HARPER'S SCIENTIFIC MEMOIRS

PRISMATIC AND DIFFRACTION SPECTRA

EDITED BY

J. S. AMES, Ph.D.

HARPER'S SCIENTIFIC MEMOIRS

EDITED BY

J. S. AMES, PH.D.

PROFESSOR OF PHYSICS IN JOHNS HOPKINS UNIVERSITY

II.

PRISMATIC AND DIFFRACTION SPECTRA

PRISMATIC AND DIFFRACTION SPECTRA

MEMOIRS BY JOSEPH VON FRAUNHOFER

TRANSLATED AND EDITED

BY J. S. AMES, PH.D.

PROFESSOR OF PHYSICS IN JOHNS HOPKINS UNIVERSITY

NEW YORK AND LONDON

HARPER & BROTHERS PUBLISHERS

1898

PREFACE

THE spectrum of the sun was first observed, in 1666, by Newton, who allowed light coming from a small round opening in a shutter to pass through a glass prism. This spectrum was most impure; and a pure spectrum was not obtained until, in 1802, Wollaston repeated Newton's experiment, replacing the round opening by a slit parallel to the edge of the prism. He observed several dark lines crossing the spectrum, which limited, as he thought, the different spectral colors. His description of this discovery is given at the end of this volume.

Working independently of Wollaston, Fraunhofer, in 1814, rediscovered the lines in the solar spectrum, which now bear his name. He at first used a slit and prism; but, later, he discovered that the same phenomena could be obtained by means of gratings made up of wires or ruled on glass. The papers of Fraunhofer in which he describes these results are printed in full in this volume. The great merit of Fraunhofer's work is the systematic, logical method by which he proceeds from investigation to investigation.

The most important contributions of Fraunhofer to the science of Spectrum Analysis are:

1. The application of the objective prism to the study of spectra of the sun, the stars, flames, etc.
2. The discovery of the principle of plane gratings.
3. The discussion of the effect upon spectra of
   a. Periodic errors in the ruling of gratings.
   b. The shape of the groove.
   c. The relative widths of opaque and open spaces.
4. The first measurements of the wave-lengths of various solar lines.

5. The discovery of the agreement in wave-length of lines in the spectra of flames and of certain stars with those in the solar spectrum.

All modern work in spectroscopy is based upon that of Fraunhofer, and a brief bibliography of the most important contributions is appended to this volume.

GENERAL CONTENTS

Determination of the Refractive and the Dispersive Power of Different Kinds of Glass, with Reference to the Perfecting of Achromatic Telescopes.

Denkschriften der königlichen Akademie der Wissenschaften zu München, V., pp. 193–226, 1817.
Schumacher's *Astronomische Abhandlungen*, Heft II., pp. 13–45, 1823.
Edinburgh Philosophical Journal, IX., X., 1823, 1824.

CONTENTS

[*Introduction is omitted.*]

It would be most advantageous if one could measure for every kind of glass the dispersion for every color; but the different colors in the spectrum have no definite limits, and so this cannot be determined immediately from the color-image. The uncertainty is so great that the experiments are useless. It could be done more exactly if colored pieces of glass or colored liquids could be found which would transmit light of only one color—*e. g.*, one which transmitted only blue, another only red, etc.; but I have not been so fortunate as to discover any such substances. With all I tried, the white light which was transmitted was broken up into all colors, the only difference being that the particular color which the glass or the liquid had was the strongest in the spectrum. Colored flames, also, which are obtained by the combustion of alcohol, sulphur, etc., do not give in their spectra homogeneous light which corresponds to their color; yet with these, as well as with oil and tallow light, and in general with light of all flames, I found in the spectra a bright, sharply defined streak in the region between the red and the green, which is in exactly the same position in all the spectra, and which will be most useful in what follows. This bright band appears to be formed by rays which are not dispersed further by the prism, and are therefore homogeneous. There is a similar streak in the green, which is, however, not so well defined and is much more feeble, so that in some cases it is recognized with difficulty; on this account it cannot be of much service.

[*The experiments on the dispersion of various prisms are omitted.*]

In order to determine more accurately the refraction of the different colored beams, partly also to see if the action of the refracting medium was the same on sunlight as on artificial light, I devised an apparatus which could be used with sunlight,

just as the one before described was with lamplight. This was soon seen to be superfluous.

In the window-shutter of a darkened room I made a narrow opening—about 15 seconds broad and 36 minutes high—and through this I allowed sunlight to fall on a prism of flint-glass which stood upon the theodolite described before. [*The dimensions are not given.*] The theodolite was 24 feet from the window, and the angle of the prism was about 60°. The prism was so placed in front of the objective of the theodolite-telescope that the angle of incidence of the light was equal to the angle at which the beam emerged. I wished to see if in the color-image from sunlight there was a bright band similar to that observed in the color-image of lamplight. But instead of this I saw with the telescope an almost countless number of strong and weak vertical lines, which are, however, darker than the rest of the color-image; some appeared to be almost perfectly black. If the prism was turned so as to increase the angle of incidence, these lines vanished; they disappear also if the angle of incidence is made smaller. For increased angle of incidence, however, these lines become visible again if the telescope is made shorter; while, for a smaller angle of incidence, the eye-piece must be drawn out considerably in order to make the lines reappear. If the eye-piece was so placed that the lines in the red portion of the color-image could be plainly seen, then, in order to see the lines in the violet portion, it must be pushed in slightly. If the opening through which the light entered was made broader, the fine lines ceased to be clearly seen, and vanished entirely if the opening was made 40 seconds wide. If the opening was made 1 minute wide, even the broad lines could not be seen plainly. The distances apart of the lines, and all their relations to each other, remained unchanged, both when the width of the opening in the window-shutter was altered and when the distance of the theodolite from the opening was changed. The prism could be of any kind of refractive material, and its angle might be large or small; yet the lines remained always visible, and only in proportion to the size of the color-image did they become stronger or weaker, and therefore were observed more easily or with more difficulty.

The relations of these lines and streaks among themselves appeared to be the same with every refracting substance; so that, for instance, one particular band is found in every case

only in the blue; another is found only in the red; and one can, therefore, at once recognize which line he is observing. These lines can be recognized also in the spectra formed by both the ordinary and the extraordinary rays of Iceland spar. The strongest lines do not in any way mark the limits of the various colors; there is almost always the same color on both sides of a line, and the passage from one color into another cannot be noted.

With reference to these lines the color-image is as shown in Fig. 5. It is, however, impossible to show on this scale all the lines and their intensities. (The red end of the color-image is in the neighborhood of A; the violet end is near I.) It is, however, impossible to set a definite limit at either end, although it is easier at the red than at the violet. Direct sunlight, or sunlight reflected by a mirror, seems to have its limits, on the one hand, somewhere between G and H; on the other, at B; yet with sunlight of great intensity the color-image becomes half again as long. In order, however, to see this great spreading-out of the spectrum, the light from the space between C and G must be prevented from entering the eye, because the impression which the light from the extremities of the color-image makes upon the eye is very weak, and is destroyed by the rest of the light. At A there is easily recognized a sharply defined line; yet this is not the limit of the red color, for it proceeds much beyond. At *a* there are heaped together many lines which form a band; B is sharply defined and is of noticeable thickness. In the space between B and C there can be counted 9 very fine, sharply defined lines. The line C is of considerable strength, and, like B, is very black. In the space between C and D there can be counted 30 very fine lines; but these (with two exceptions), like those between B and C, can be plainly seen only with strong magnification or with prisms which have great dispersion; they are, moreover, very sharply defined. D consists of two strong lines which are separated by a bright line. Between D and E there can be counted some 84 lines of varying intensities. E itself consists of several lines, of which the one in the middle is somewhat stronger than the rest. Between E and *b* are about 24 lines. At *b* there are 3 very strong lines, two of which are separated by only a narrow bright line; they are among the strongest lines in the spectrum. In the space between *b* and F there can be counted about 52 lines; F is

fairly strong. Between F and G there are about 185 lines of different strengths. At G there are massed together many lines, among which several are distinguished by their intensity. In the space between G and H there are about 190 lines, whose intensities differ greatly. The two bands at H are most remarkable; they are almost exactly equal, and each consists of many lines; in the middle of each there is a strong line which is very black. From H to I the lines are equally numerous. In the space between B and H there can be counted, therefore, about 574 lines, of which only the strong ones appear on the drawing. The distances apart of the strongest lines were measured by the theodolite and transferred according to scale directly to the drawing; the weak lines, however, were drawn in, without exact measurement, simply as they were seen in the color-image.

I have convinced myself by many experiments and by varying the methods that these lines and bands are due to the nature of sunlight, and do not arise from diffraction, illusion, etc. If light from a lamp is allowed to pass through the same narrow opening in the window-shutter, none of these lines are observed, only the bright line R [*referred to before*], which, however, comes exactly in the same place as the line D (Fig. 5), so that the indices of refraction of the rays D and R are the same. The reason why the lines fade away, or even entirely vanish, when the opening at the window is made too wide is not difficult to give. The stronger lines have a width of from five to ten seconds; so, if the opening of the window is not so narrow that the light which passes through can be regarded as belonging to one ray, or if the angular width of the opening is much more than that of the line, the image of one and the same line is repeated several times side by side, and consequently becomes indistinct, or vanishes entirely if the opening is made too wide. The reason why the lines and bands are not seen when the prism is turned, unless the telescope is made shorter or longer, will be seen from the following considerations:

It is only when rays fall upon a prism in such a way that the angle of incidence equals that of emergence that they proceed, so far as divergence is concerned, just as they fall upon the prism; if the angle of incidence is greater, the rays, after refraction through the prism, diverge from a more remote point; if the angle is smaller, the rays diverge from a nearer point.

The reason is that the rays which pass through near the edge of the prism have a shorter path through the prism to traverse than those which pass through farther from the edge. This does not change, it is true, the angle of the refracted rays; but the sides of the triangle for the emerging rays become greater in the one case, less in the other. This difference should vanish when the rays fall upon the prism parallel to each other, which is in accord with experiment. Since the violet rays through the objective of the theodolite telescope have a shorter focal length than the red rays, it is evident why the eye-piece must be displaced in order to see plainly the lines in the different colors.

Since the lines and bands in the color-image have only a very small width, it is evident that the apparatus must be most perfect in order to avoid all aberrations which could make the lines indistinct or entirely scatter them. The faces of the prism must therefore be perfectly plane. The glass to be used in such prisms should be entirely free from waves and streaks; with English flint-glass, which is never free from streaks, only the stronger lines are seen. Common glass and English crown-glass contain many streaks, even if they are not visible to the naked eye. If one does not possess a prism of perfect flint-glass, it is better to choose a fluid which has a great dispersive power—*e. g.*, aniseed oil—in order to see the lines; yet in this case the prismatic vessel must have faces which are perfectly plane parallel. With all prisms the faces should make an angle of 90°, or nearly so, with the base; this must be placed horizontal, in front of the telescope, if the axis of the latter is horizontal. The narrow opening through which the light enters must be exactly vertical, etc. It is easy to understand why the lines become indistinct if one or the other of these precautions is neglected.

Since the lines and bands are seen in the color-image of every refracting medium of uniform density, I have used them in order to measure the refraction of a medium for each colored ray; and this could be done with great accuracy, because the majority of the lines are sharply defined. Since with refracting media which have feeble dispersion or with prisms of small angles it is only with difficulty that the fine lines can be recognized even with strong magnification, I selected in these experiments, for all refracting media, the stronger lines, *viz.*,

B, C, D, E, F, G, and H. The lines at *b* I did not take, because it is too near F; and I tried to come more in the middle between D and F. Since the eye-piece must be displaced in order to see distinctly the lines in the different colors, no large arcs such as BH could be measured, *only the small ones* such as BC, CD, etc.

[*Ten pages, containing descriptions of experiments on different kinds of glass, are omitted.*]

On seeing the number of lines and bands in the spectrum of sunlight, one can perhaps with difficulty avoid the conjecture that diffraction at the narrow opening in the window-shutter has something to do with these lines, although the experiments which have been described do not in the least indicate this, but, on the contrary, refute it. Partly in order to be perfectly sure on this point, partly also in order to make further observations, I changed the experiment in the following manner :

If sunlight coming from a small *round* opening, 15 seconds in diameter, in the window-shutter is allowed to fall upon a prism placed in front of the theodolite telescope, it is clear that the color-image which is seen through the telescope can have only an inappreciable width, and therefore will form only a line ; in a colored line, however, no fine cross-lines can be seen. In order to see the many lines in this color-image, all that is necessary is to make the spectrum broader without altering its length in the least. I accomplished this by placing against the objective a piece of glass which was plane on one side and curved on the other, so as to be a portion of a cylinder of large diameter. The axis of the cylinder was placed parallel to the base of the prism; consequently, the length of the spectrum could not be changed, and only its breadth was increased. With this arrangement I recognized again in the spectrum all the lines exactly as they are seen when the light comes through a long, narrow opening.

I applied this form of apparatus at night-time to observe Venus directly, *without making the light pass through a small opening;* and I discovered in the spectrum of this light the same lines as those which appear in sunlight. Since, however, the light from Venus is feeble in comparison with sunlight reflected from a mirror, the intensity of the violet and the extreme red rays are very weak ; and on this account even the stronger lines in both these colors are recognized only with

difficulty, but in the other colors they are very easily distinguished. I have seen the lines D, E, *b*, F perfectly defined (Fig. 5), and have even recognized that *b* consists of two lines, one weak and one strong; but the fact that the stronger one itself consists of two I could not verify owing to lack of light. For the same reason the other finer lines could not be distinguished satisfactorily. I have convinced myself by an approximate measurement of the arcs DE and EF that the light from Venus is in this respect of the same nature as sunlight.

With this same apparatus I made observations also on the light of some fixed stars of the first magnitude. Since, however, the light of these stars is much weaker than that of Venus, it is natural that the brightness of the spectrum should be much less. In spite of this I have seen with certainty in the spectrum of Sirius three broad bands which appear to have no connection with those of sunlight; one of these bands is in the green, two are in the blue. In the spectra of other fixed stars of the first magnitude one can recognize bands; yet these stars, with respect to these bands, seem to differ among themselves. Since the objective of the telescope has an aperture of only 13 lines*, it is clear that these observations can be repeated with much greater accuracy. I intend to repeat them with suitable alterations, and with a larger objective, in order to induce, perhaps, some skilled investigator to continue the experiments. Such a continuation is all the more to be desired, because the experiments would serve at the same time for the accurate comparison of the refraction of the light of the fixed stars with that of sunlight.

The light of electricity is, with respect to these lines and bands, markedly different from sunlight, and also from the light of flames. Several lines are found in the spectrum of this light; some are very bright, and among these one in the green is almost dazzlingly bright in comparison with the rest of the spectrum. There is another line in the orange which is not quite so bright; it appears to have the same color as the bright line in the spectrum of lamplight. If, however, the angle of refraction is measured, it is found that its light is refracted much more—about as much as the yellow rays of lamplight. Near the red end of the spectrum there is a line

[* 1 *centimetre equals* 4.43296 *lines.*]

which is not very bright; its light is refracted, so far as I could be sure, exactly the same as that of the bright line of lamplight. In the rest of the spectrum four other lines can be recognized easily.*

If lamplight is allowed to pass through a very narrow opening of 15 to 30 seconds' width and then to fall upon a strongly dispersive prism placed in front of a telescope, it is seen that the reddish-yellow bright line of this spectrum consists of two very fine bright lines which in intensity and distance apart are like the two dark lines D (Fig. 5). But, regardless of the width of the slit, if the point of the flame and the lower blue end are covered, and so only the brightest portion of the flame exposed, the reddish-yellow lines of the spectrum appear less bright, and therefore are recognized with more difficulty. These lines, consequently, seem to be formed mainly by the light from the ends of the flame, especially by that from the lower one.

In the spectrum of the light caused by the combustion of hydrogen and alcohol, the reddish-green line is very bright in comparison with the other portions. When sulphur is burned it is seen with great difficulty.

I intend to repeat these experiments which have reference to the perfecting of achromatic telescopes, using a new instrument by means of which I hope to obtain at least twice the accuracy. I shall be able also to make new experiments with this instrument for which my present one is not adapted, and which will be of interest perhaps for practical optics.

In all my experiments I could, owing to lack of time, pay attention to only those matters which appeared to have a bearing upon practical optics. I could either not touch other questions, or at most not follow them very far. Since the path thus traced in optical experiments seems to promise to lead to interesting results, it is greatly to be desired that skilled investigators should devote attention to it.

* In order to use the electric light I placed two conductors one-half an inch apart, one being joined to an electrical machine, the other to the earth, and connected them with a fine glass fibre. The light then appeared to pass continuously over the fibre, and the latter formed a fine luminous line.

New Modification of Light by the Mutual Influence and the Diffraction of the Rays, and the Laws of this Modification.

Denkschriften der königlichen Akademie der Wissenschaften zu München, 1821, 1822, VIII., 1–76.
Schumacher's *Astronomische Abhandlungen,* II., 46–112, 1823.

CONTENTS

New Modification of Light by the Mutual Influence and the Diffraction of the Rays, and the Laws of this Modification.

All experiments in which the eye of the investigator is provided with good optical instruments are distinguished, as is well known, by a high degree of precision; and some of the most important discoveries could not have been made without these instruments. Up to the present time, in experiments on diffraction there has been no instrument, except a magnifying-glass, which could be used with profit; and this may perhaps be one of the reasons why in this field of physical optics we are so backward, and why we know so little of the laws of this modification of light. Since at small angles of inclination refraction and reflection of light are altered by diffraction, and since in many other cases diffraction plays an important part, which may often be unnoticed, it is most to be desired that these laws should be exactly known; and this is specially so because a knowledge of them makes the nature of light itself better known at the same time.

If sunlight is admitted into a darkened room through a small opening and falls upon a dark screen some distance away, which has a narrow aperture, and if the light which passes through this slit is allowed to fall upon a white surface or a piece of ground-glass placed a short distance behind the screen, one sees, as is well known, that the illuminated portion of the white surface is larger than the narrow slit in the screen, and that it has colored edges—in short, that the light through the slit is inflected or diffracted. The narrower the openings, so much the greater is the inflection. The shadow of every body which is placed in a beam of sunlight entering a darkened room through a small opening is bounded by fringes of color which are, moreover, for any given distance of the surface on which the shadow is received, of the same size for bodies of all

kinds of matter. The shadow of a narrow object, such as a hair, has, in addition to the outer fringes, others within the shadow, which change with the thickness of the hair, but in other respects are similar to the outer ones.* Since the colored fringes are very small, and since most of the light is lost through absorption at the surface on which the shadow is cast, no great accuracy could be expected with the methods which have been used up to this time to observe diffraction phenomena; and this is all the more true because by these methods it is impossible to measure the angles of inflection of the light which alone can make us acquainted with the laws of diffraction. Up to the present, these angles from which the path of the diffracted light can be learned have been calculated from the dimensions of the colored bands and their distance from the diffracting body; but assumptions have been made which, as we shall see, do not agree with the truth, and which, therefore, give false results.

The number of different optical phenomena has become in our time so great that caution must be taken so as to avoid being deceived, and also to refer the phenomena always to the simple laws. This is more necessary in the case of diffraction, as we shall see, than in all the other phenomena. I shall, therefore, report the experiments which I have made for the determination of the laws of diffraction of light in an order which is different from that in which I actually performed them, by which procedure many experiments become superfluous and a better understanding will be reached.

Diffraction of Light through a Single Opening.

In order to receive in the eye all the light diffracted through a narrow opening, and to see the phenomena strongly magnified; still more, in order to directly measure the inflection of the light, I placed in front of the objective of a theodolite-telescope a screen in which there was a narrow vertical opening which could be made wider or narrower by means of a screw. By means of a heliostat I threw sunlight into a darkened room

* All that is known on the subject of diffraction may be found in Biot's "Traité de Physique, Exp. et Math.," T. 4, p. 743; and in the *Göttinger Commentaren*, Vol. IV., p. 49.

through a narrow slit so that it fell upon this screen, through whose opening the light was therefore diffracted. I could then observe through the telescope the phenomena produced by the diffraction, magnified, and yet seen with sufficient brightness; and at the same time I could measure the angles of inflection of the light by means of the theodolite.

The colors which are produced by the diffraction of light through a single opening are arranged in an order similar to that of the colors of Newton's rings, which are produced by the contact of two slightly convex pieces of glass; with this difference, that with the latter a black spot is seen in the centre, while it is not with the former. Fig. III., Table I., will help the description. If the telescope of the theodolite is so adjusted that on removing the screen which has the diffraction-slit the slit at the heliostat is focused on the micrometer cross-hairs, and if then the screen—whose slit must be very narrow—is placed in front of the objective, there will be seen in the centre of the field a white band $L^I L^I$; and the cross-hairs will be in the middle of this band at K. This band becomes yellow near each side, and finally red. In the space $L^I L^{II}$ there is a vivid color-spectrum, which is indigo near L^I, then blue, green, yellow, and near L^{II} red. The color-spectrum in the space $L^{II} L^{III}$ is much less intense than that in $L^I L^{II}$; the arrangement of its colors is as follows: Near L^{II} blue, then green, yellow, and near L^{III} red. The spectrum in the space $L^{III} L^{IV}$ is still weaker than the last; near L^{III} it is green; near L^{IV}, red. There then follow a great number of spectra which grow continually weaker until they can be no longer distinguished, and then can be seen only a horizontal strip of light which is, however, stretched out through a great distance. The spectra just described are exactly the same on the two sides of K—*i. e.*, they are symmetrical. The transitions from one color into another are not sharply defined, but imperceptible, and the same thing is true of the spectra.

The instrument with which I observed and measured the angles is essentially a 12-inch repeating theodolite whose verniers read to 4″. In the middle of the circle and above it there is a plane horizontal disk 6 inches in diameter which turns on its axis, and whose centre lies exactly on the axis of the theodolite. It has its own divisions, which read to 10″. At the middle of this disk is placed the screen through which the

light is diffracted, and which therefore stands in the axis of the theodolite, so that the corrections to the angles as measured, which otherwise would have to be made, owing to the distance of the diffracting body from the axis, are not necessary. The divisions marked on the disk serve to measure, if it is necessary, the angle of incidence of the light, etc. Beyond the disk, at a distance of 3¼ inches* from the centre, the telescope begins. Its objective has an aperture of 20 lines and a focal length of 16.9 inches, and it is clamped to the alidade of the 12-inch circle and suitably counterbalanced. The axis of the telescope is parallel to the plane of the circle and accurately horizontal. I used a magnifying power of 30 and 50. The whole instrument is isolated from the floor. In the prolongation of the optical axis, 463½ inches from the middle of the theodolite, is placed the heliostat, whose hour motion can be regulated by a screw and a rod which is attached to it and which reaches to the theodolite, so that one can at will increase or decrease the intensity of the sunlight. The opening at the heliostat is vertical, 2 inches long, and can be made wider or narrower. I ordinarily used it with a width of only 0.01 to 0.02 inch.

I measured the width of the opening in the screen by means of an achromatic microscope which I devised for this purpose, because this quantity must be known most exactly. At the foot of this microscope there is a sliding-piece which can be moved in one direction by a fine screw having nearly 88 turns to a Paris inch; the screen is so fastened to this sliding-piece that its opening, which is to be measured, comes vertically above the screw. There are cross-hairs in the eye-piece of the microscope, which can be seen at the same time as the object viewed. By means of the screw which moves the sliding piece, first one edge and then the other of the object is made to coincide with one edge of the cross-hair, and the position of the screw is read each time. The difference in the readings is the diameter of the object, in terms of the pitch of the screw, independently of the construction of the optical parts of the microscope, the magnification, etc. Since the head of the screw is divided by a vernier into 1000 parts, it is possible to measure the diameter of a sharply defined object to at least 0.00002 of an inch; in many cases to 0.00001. I generally used an objec-

* [1 *Paris inch* = 2.70700 *cm.* = 1.06577 *inches.*]

tive with which the microscope magnifies the diameter of the object 110 times.

Since it is impossible to find a fixed point of reference in the color-spectrum arising from diffraction through a single narrow opening, I took, in order to measure the angles of deflection, the transition from one spectrum into another—that is, L^{I}, L^{II}, L^{III}, etc., or the red end of each spectrum. I measured the distances $L^{I}L^{I}$, $L^{II}L^{II}$, etc., repeating at least three times. The halves of these are, therefore, the deflections from the centre—that is, KL^{I}, KL^{II}, etc. I shall designate by L^{I}, L^{II}, etc., the angles of these deflections. All spectra which are due to diffraction through a single opening I shall call spectra of the *first class*, merely to distinguish them from other kinds which will be discussed later. The following table contains the angles of inflection of light through openings of different widths. I designate this width by γ; it is expressed always in fractions of a Paris inch. I call the arithmetical mean of L^{I}, $\frac{L^{II}}{2}$, $\frac{L^{III}}{3}$, etc., L.

No.	γ	L^{I}	L^{II}	L^{III}	L^{IV}	L	$L\gamma$
							0.0000
1	0.11545	37″.58	1′ 15″.5	1′ 53″	....	37″.66	210
2	0.06098	1′ 11″.6	2′ 22″.7	3′ 31″.7	4′ 44″.7	1′ 11″.17	210
3	0.03690	1′ 57″.1	3′ 53″.3	5′ 48″.3	....	1′ 56″.6	209
4	0.02346	3′ 4″	6′ 7″.7	9′ 16″.3	....	3′ 4″.43	210
5	0.01237	5′ 48″.5	11′ 38″	17′ 26″.5	23′ 14″.7	5′ 48″.7	209
6	0.01210	6′	12′ 1″	18′ 14″	24′ 9″	6′ 1″.84	212
7	0.01020	6′ 56″	13′ 56″	20′ 54″	....	6′ 57″.3	206
8	0.00671	11′ 6″	22′ 12″.7	33′ 14″	44′ 35″	11′ 6″.4	217
9	0.00642	11′ 11″	22′ 18″	33′ 43″	44′ 58″	11′ 12″.2	209
10	0.00337	21′ 3″	42′ 16″	1° 4′	....	21′ 10″.3	207
11	0.00308	23′ 31″	47′ 6″	1° 10′ 43″	....	23′ 32″.7	211
12	0.00218	33′ 30″	1° 7′ 40″	......	....	33′ 40″	213
13	0.00215	35′ 24″.7	1° 10′ 16″	......	....	35′ 17″	220
14	0.00114	1° 4′ 53″	......	......	....	1° 4′ 53″	215

The angles recorded in this table are all given exactly as I observed them, without any corrections, and it is not difficult, therefore, to determine the limits of accuracy. Since the points of transition from one spectrum into another are not sharply defined, recourse must be had, within certain limits, to estimation. In long spectra—*i.e.*, when the opening in the screen is narrow—these limiting positions lie farther apart, and the angles will not agree among themselves as well as in the case of a large opening in the screen and small spectra. The

relative accuracy is, however, nearly the same. Within the limits of error, the following conclusions may be drawn from the table:

With single openings of different widths the angles of inflection of the light are inversely proportional to the widths of the opening.

In light diffracted through a narrow opening the distances from the centre to the red rays of the various spectra on both sides form an arithmetical series in which the common difference is equal to the first term.

It will appear from later experiments that this law holds for the other colored rays, and that it is true for the spectra which are far from the axis as well.

In the general case, for any width of opening, measured in fractions of a Paris inch and designated by γ, where L^{I}, L^{II}, L^{III}, etc., are the arcs for radius 1, we have:

$$L^{I} = \frac{0.0000211}{\gamma},$$

$$L^{II} = 2 \cdot \frac{0.0000211}{\gamma},$$

$$L^{III} = 3 \cdot \frac{0.0000211}{\gamma}, \text{ etc.}$$

In order to see whether the spectra which arise from diffraction consist of homogeneous light, I fastened in front of the eye-piece of the theodolite-telescope [*i.e., between the eye-piece and the eye of the observer*] a small flint-glass prism of about 20°, in such a manner that the axis of the prism was horizontal and the edge pointed down. If, with an eye-piece like this, one views a homogeneous spectrum such as is obtained when a good prism is placed in front of the objective, the cross-hairs of the eye-piece will be seen in every color. If, however, there is no homogeneous light in the field of view, the horizontal thread will disappear. The explanation is not difficult. If the spectra produced by diffraction through a single opening are brought into the field, no trace of the horizontal thread is seen in the first and second spectra; in the third it is just perceptible; in the fourth it is more definite, although still indistinct. This indistinctness becomes

gradually less in the succeeding spectra, so that far from the middle of the field the horizontal thread can be seen fairly defined. Accordingly, the spectra near the axis consist of non-homogeneous light; those farther away become more and more homogeneous.

The lower horizontal red end of the first spectrum appears blue when viewed through the prism; the upper blue end of this spectrum, however, appears red, which also proves that this spectrum consists of non-homogeneous light; for in a spectrum formed by a prism no blue light can be produced out of red, nor red from blue. Since the light is refracted through the eye-piece prism, and since rays are refracted differently—*e.g.*, the blue more than the red—it, therefore, happens that, when there is in the field of view a homogeneous [*i.e.*, *pure*] spectrum, which without the prism would be horizontal, the thread of the cross-hair which would have been horizontal is no longer so, but is lower at the end where it points towards the more refrangible rays, and higher at the other; it, therefore, has an inclined position which may be easily noticed. Since in the spectra formed by diffraction those which are far from the axis overlap, and since a portion of each spectrum encroaches upon the one which precedes and the one which follows, as is apparent from the second of the above laws, the inclined position of the cross-hair serves to convince one of their existence and furnishes a method of counting them. For if there are several overlapping spectra in the field of view, there will be seen as many oblique cross-hairs as there are spectra. I shall return to this matter later.

[*The case is then discussed when the two halves of the screen forming the slit are no longer in the same plane, but one is displaced normally with reference to the other.*]

In all the above experiments the light at the heliostat fell upon a narrow vertical opening, so that only one beam of light entered, or, in other words, so that the light apparently came from a luminous line. The reason for this is easily understood; in every other case each beam would form its own spectrum, and there would be as many of these overlapping as there were beams. If, for instance, the light came from a luminous surface whose angular width was greater than that of the spectra, it would be impossible to distinguish any spectra through a narrow opening in the screen; because the rays

coming from the right-hand side of the luminous surface would produce red in the same place where the rays from the left side would produce blue, etc.; and the light would be all confused, and consequently white again. Since, however, light is diffracted by every narrow opening, one might imagine that the light falling upon the slit in the screen had already been modified by diffraction at the slit near the heliostat. Although this doubt disappears, either by a consideration of the diameter of the sun or on certain other grounds, nevertheless I have investigated the subject experimentally. All that is necessary is that the light should come as if from a luminous line. I prepared, therefore, a lens, 2 inches long, $\frac{1}{3}$ inch wide, which was plane upon one side, and upon the other was a portion of a cylinder of 0.66 inch diameter. [*Parallel*] light falling upon this lens proceeds after refraction as if it came from a line at a distance of 0.62 inch from the lens. The opening at the heliostat was made $\frac{1}{4}$ inch wide, and the cylindrical lens was placed in front of it. If the path of the light through the lens is traced, it is seen that no light passing around the edge of the slit at the heliostat can fall upon the screen which is in front of the theodolite-telescope, and consequently no diffracted light can reach it. Using this cylindrical lens, both the spectra due to diffraction through a small opening and their dimensions are exactly the same as they were when the narrow slit was used at the heliostat.

By means of a screen which contains a long, narrow opening, light is diffracted in one direction only; by my screen this direction was horizontal, because the opening was vertical. A screen containing an opening which is not narrow, but is, for instance, as high as it is broad, will produce diffraction in a vertical direction also. It is easily understood that in this case the incident light must not come from a luminous line, because the diffraction in a vertical direction would not be observed, for the same reasons as I have given above. The light must come through an opening at the heliostat, which is as broad as it is high. I, therefore, allowed the light, as a rule, to pass through a round opening 0.04 to 0.08 inch in diameter. If a round opening is used, and if the screen with a long vertical opening is placed in front of the theodolite-telescope, the spectra, as can be easily predicted, have a very small height, but in a horizontal direction they are exactly as they were when the light

passed through a long vertical opening at the heliostat. There is seen, therefore, only a horizontal line, in which the colors alternate as described above, and which is narrower in proportion as the round opening at the heliostat is made smaller. This opening should not be made too small, however, because if the light is to be diffracted in other directions also, there would not be sufficient brightness.

If the light passes through a round opening at the heliostat, and if there is placed in front of the theodolite-telescope a screen containing a rectangular opening, which has perfectly straight sides and sharp corners, and which is, *e.g.*, as high as it is wide, a colored cross will be seen in the telescope, in which the colors alternate vertically as well as horizontally in the same way as they did in light diffracted by a long, narrow opening. In the corners of this colored cross there are seen still feebler color-spectra, a, b, c, d, Table II., Fig 2. [*This figure is omitted.*] The explanation of these spectra which are seen only in the corners will be apparent from experiments which will be discussed further on. The dimensions of the colors which form the cross are the same as when a long, narrow opening of the same width is used: *viz.*, $L^{I}=\frac{0.0000211}{\gamma}$, $L^{II}=2.\ \frac{0.0000211}{\gamma}$, etc., in both a vertical and a horizontal direction. If the rectangular opening in the screen is not as broad as it is high, the vertical spectra of the cross have a different width from the horizontal ones; and the weak subsidiary spectra in the corners are altered accordingly. By means of a screen, then, the width of whose rectangular opening is less than its height, there is produced a colored cross whose vertical arm is composed of spectra smaller than the horizontal ones, in the inverse ratio of the height to the width.

[*Diffraction through a circular opening is then discussed.*]

MUTUAL ACTION OF A GREAT NUMBER OF DIFFRACTED BEAMS.

In order to make a great number of beams, diffracted exactly alike, fall upon the whole surface of the theodolite-objective, I stretched upon a frame a great many wires of the same thickness, parallel to each other and at the same distances apart; the light was then diffracted through the inter-

vening spaces. In order to be sure that the wires were exactly parallel and at equal distances from each other, I cut on two opposite ends of a rectangular frame a fine screw-thread, which had 169 turns, nearly, to a Paris inch; I stretched the wires in the spirals of this screw, and I could then be sure that they were exactly parallel and at equal distances apart.

Through a vertical opening at the heliostat, 2 inches high and 0.01 inch wide, I directed an intense beam of sunlight upon the objective of the theodolite-telescope, and placed the grating at the middle of the theodolite-disk. The grating consisted of about 260 parallel wires, which were 0.002021 inch thick and whose edges were 0.003862 inch apart. I took care to see that no light except that coming through the grating fell upon the objective. Since the small openings between the threads diffract the light, all the light which fell upon the objective was diffracted to the same extent. I was most surprised to observe that the phenomena seen through the telescope when the grating was used were entirely different from those observed by diffraction through a single opening. The opening at the heliostat is seen exactly as it would be through the telescope if the grating were away; and at some distance from this image, on both sides, are seen a great number of colored spectra, which are exactly like those seen through a good prism; they become longer the farther they are from the axis, but they decrease in intensity. Fig. 1, Table I., represents some of these spectra. In A is seen the opening at the heliostat, colorless and sharply defined, just as it would appear if the grating were removed. On both sides of A the phenomena are perfectly symmetrical. If the apparatus is well made, there is no light in the space AH^{I}. The first spectrum is in the space $H^{I}C^{I}$; H^{I} is the violet end, C^{I} the red. There is no light in the space $C^{I}H^{II}$. The second spectrum is in the space $H^{II}C^{II}$; it is twice as long as the first, and the order of the colors is the same; it is somewhat less intense than the first. In the space between C^{II} and F^{IV} comes the third spectrum; a portion of its violet rays, however, overlaps the red of the second, and the red of the third overlaps the blue of the fourth. The intensity of the third spectrum is again less than that of the second. Between F^{IV} and D^{IV} is the fourth spectrum, whose blue end overlaps the third, and whose red end overlaps the fifth spectrum. Other spectra follow, which become gradually weaker, and of which

13 on each side of A can be counted without difficulty if the apparatus is perfected. One can convince himself easily of the existence of a still greater number; and the only reason why they cannot be counted is because they become longer and longer, and consequently overlap more and more.

If the eye-piece of the telescope is so focused that, if the grating were away, the opening at the heliostat would appear perfectly sharp, there are seen in the spectra produced by the grating the same lines and streaks which I discovered in the spectrum of sunlight produced by a good prism.* This is of special interest, because it makes it possible, as we shall see, to determine with a great degree of accuracy the laws of this modification of light produced by the mutual action of a great number of diffracted beams. In the drawing I have represented in each spectrum only the strongest of these lines, those with which we shall have to do hereafter; there is seen, however, especially in the longer spectra, a great number of lines, exactly as with a prism. The relative intensity of the lines and their arrangement are the same as with a prism. It is only with reference to the relative spaces which in any one spectrum are occupied by different colors that there is a striking difference between the spectra produced by the grating and the prism. On this account, and because with some kinds of gratings the spectra are very small, one must be well acquainted with the lines formed by a prism in order to know at once, in spectra of every size, with which bands or lines he has to do. This is all the more necessary because in the grating-spectra which are far from the axis the lines are superimposed.

I shall call these spectra which are produced by a grating of parallel threads "*perfect spectra of the second class,*" so as to distinguish them from others which arise from the mutual action of a smaller number of beams, and in which the bands and lines are not seen. These spectra have different properties, and I shall call them "*imperfect spectra of the second class.*"

In order to vary the conditions of the phenomena as much as possible, I made gratings of wires of different thicknesses and

* I have described them in a memoir printed in the *Denkschriften der k. b. Akademie der Wissenschaften* for the year 1814–15, which bears the title, "Determination of the refractive and dispersive power of different kinds of glass with reference to the perfecting of achromatic telescopes." [*This volume, page* 2.]

with different spaces. For this purpose I made a fine screw on which there were nearly 340 turns to the inch. I also drew parallel straight lines at equal intervals on a piece of plane glass covered with gold-foil, through which spectra were obtained exactly as they were through wire gratings.

The size of the spectra of the second class, which are produced by a grating, does not depend upon the width of the spaces nor upon the thickness of the wires, but only upon the sum of the two, or, what is the same thing, upon the distance apart of the centres of the spaces. The less this distance is, the larger are the spectra. The finer, therefore, the screw in whose grooves the thread is stretched, the larger is the spectrum. Consequently, for these spectra of the second class it is entirely immaterial what the thickness of the wire or the width of the openings is. A hair, silver wire, or gold wire can be stretched in the screw-threads; the kind of matter is of no importance. Care must be taken, however, to have the thread of uniform thickness, and particularly to have it stretched straight in order that the openings may be of the same width throughout their entire length. If wire is used, great care must be taken in stretching it over the screws, because it bends so easily. It is difficult to use hairs, because they rarely have a uniform thickness.

If the grooves of the screw upon which the threads are wound are somewhat large—*i.e.*, if the centres of the openings lie far apart—the spectra are small, as noted above; and consequently all come in a small space. If with these coarse grooves the wires are thick, and therefore the width of the openings are correspondingly small, one can see that where the perfect spectra of the second class cease—or, better, become weaker—another kind of spectrum begins. These are much longer; and the lines and bands which occur in the prismatic spectrum are not seen in them. They change only with the width of the grating-openings, and behave like spectra of the first class, which are produced by diffraction through a single narrow opening; and, therefore, I designate them by L^{I}, L^{II}, etc.

We shall see that the spectra of the first class are visible with nearly all perfect gratings regardless of whether the wires are wound on fine or on coarse grooves. It often happens that some of the spectra of the first class coincide with the spec-

tra of the second class, and so modify their intensity. We shall see the connection between these wonderful phenomena from the experiments.

If, when a grating is used, the prism described above in the case of diffraction through a single opening is placed, as there described, in front of the eye-piece, we see that the perfect spectra of the second class consist of homogeneous light, and that beginning with the third, owing to the increase in their length, they overlap at the places of transition from one spectrum into another. On account of the unequal refraction of the different colored beams through the eye-piece prism, the overlapping spectra will be in part separated and appear as is shown in Fig. 4, Table II. [*This figure is omitted.*] We see, for instance, the red end of the third spectrum at C^{III}, and the lines which belong to this color may be definitely recognized; likewise we see the violet end of the third spectrum at H^{III}, and the lines belonging to it. The same thing is true of spectra which lie farther from the axis. Since spectra become longer the farther they are from the axis A, and since for an eye-piece prism of definite angle the height $C^{I}d$ [*the difference in refraction, i.e., the dispersion of C and H by the prism*] is the same for all spectra, the slant of the upper and lower edges of spectra far from the centre must be less steep than that of those near the centre. As will be seen from the observations, glass has an action upon rays of different color different from that of a grating in air. This is the reason why the lower and upper edges of the spectra viewed through the prismatic eye-piece are seen to be non-rectilinear. The horizontal cross-hair of the micrometer is seen well defined in all perfect spectra of the second class, and may be used here also to count the number of the spectra far from the centre, which, on account of their great length and small inclination, are distinguished with difficulty, even with the eye-piece prism.

If the light passes through a cylindrical lens at the heliostat, the phenomena produced by the grating remain the same as when the light passed through a narrow opening.

In the experiments which follow I have used the same system of names for the different lines of the spectra as I did in the prismatic spectrum, viz., C, D, E, F, G, H.* For the first

* [*A summary of the preceding paper is given in the note, which is omitted.*]

spectrum I shall use the symbols C^{I}, D^{I}, E^{I}, etc.; for the second, C^{II}, D^{II}, E^{II}, etc. I shall call the thickness of the wire δ, and the width of an opening γ. These quantities are given in terms of a Paris inch. The arithmetical mean of, *e. g.*, C^{I}, $C^{II}/_2$, $C^{III}/_3$, etc., I call C; that of D^{I}, $D^{II}/_2$, $D^{III}/_3$, etc., D, etc. By means of the theodolite I have measured, at least six times, the angular distance of the two symmetrical spectra for each color, or, more often, that of the lines named above. Since the spectrum lines are sharply defined, great accuracy can be obtained with perfect gratings. I give all the angles, exactly as I measured them, without any correction. The grating was always placed at the middle of the horizontal disk of the theodolite. All angles, *e. g.*, C^{I}, D^{I}, E^{I}, etc., are the simple distances from the axis A. In the products $(\gamma+\delta)C$, etc., I have used the sine of the angles. However, with such small angles as these it is entirely immaterial which is used, the sine or the arc.

GRATING No. 1.

γ = 0.000628	δ = 0.001324
B^{I} = 44′ 45″	E^{III} = 1° 42′ 42″.7
C^{I} = 42′ 42″.3	E^{IV} = 2° 16′ 59″.7
C^{II} = 1° 25′ 25″	F^{I} = 31′ 32″.6
D^{I} = 38′ 19″.3	F^{II} = 1° 3′ 10″
D^{II} = 1° 16′ 38″	F^{III} = 1° 34′ 44″
D^{III} = 1° 55′	G^{I} = 27′ 57″.3
D^{IV} = 2° 33′ 14″.7	G^{II} = 55′ 51″.7
E^{I} = 34′ 12″.6	H^{I} = 25′ 42″.3
E^{II} = 1° 8′ 28″.3	H^{II} = 51′ 31″.7
B = 44′ 45″	F = 31′ 34″.1
C = 42′ 42″.4	G = 27′ 56″.5
D = 38′ 19″.2	H = 25′ 44″
E = 34′ 14″	
$(\gamma+\delta)$ B = 0.00002541	$(\gamma+\delta)$ F = 0.00001792
$(\gamma+\delta)$ C = 0.00002425	$(\gamma+\delta)$ G = 0.00001587
$(\gamma+\delta)$ D = 0.00002176	$(\gamma+\delta)$ H = 0.00001461
$(\gamma+\delta)$ E = 0.00001944	

[*In the following tables only the results are given.*]

GRATING No. 2.

γ = 0.001112	δ = 0.001817
$(\gamma+\delta)$ B = 0.00002541	$(\gamma+\delta)$ F = 0.00001795
$(\gamma+\delta)$ C = 0.00002426	$(\gamma+\delta)$ G = 0.00001587
$(\gamma+\delta)$ D = 0.00002177	$(\gamma+\delta)$ H = 0.00001464
$(\gamma+\delta)$ E = 0.00001946	

GRATING No. 3.

$\gamma=0.000972$ $\quad\delta=0.001964$
$(\gamma+\delta)\,C=0.00002425$ $\quad(\gamma+\delta)\,F=0.00001788$
$(\gamma+\delta)\,D=0.00002176$ $\quad(\gamma+\delta)\,G=0.00001580$
$(\gamma+\delta)\,E=0.00001942$ $\quad(\gamma+\delta)\,H=0.00001450$

GRATING No. 4.

$\gamma=0.000549$ $\quad\delta=0.003359$
$(\gamma+\delta)\,B=0.00002542$ $\quad(\gamma+\delta)\,F=0.00001794$
$(\gamma+\delta)\,C=0.00002426$ $\quad(\gamma+\delta)\,G=0.00001586$
$(\gamma+\delta)\,D=0.00002178$ $\quad(\gamma+\delta)\,H=0.00001457$
$(\gamma+\delta)\,E=0.00001947$

GRATING No. 5.

$\gamma=0.003862$ $\quad\delta=0.002021$
$(\gamma+\delta)\,C=0.00002423$ $\quad(\gamma+\delta)\,F=0.00001786$
$(\gamma+\delta)\,D=0.00002174$ $\quad(\gamma+\delta)\,G=0.00001578$
$(\gamma+\delta)\,E=0.00001938$ $\quad(\gamma+\delta)\,H=0.00001420$

GRATING No. 6.

$\gamma=0.001036$ $\quad\delta=0.006759$
$(\gamma=\delta)\,C=0.00002422$ $\quad(\gamma+\delta)\,F=0.00001785$
$(\gamma+\delta)\,D=0.00002175$ $\quad(\gamma+\delta)\,G=0.00001591$
$(\gamma+\delta)\,E=0.00001942$ $\quad(\gamma+\delta)\,H=0.00001451$

GRATING No. 7.

$\gamma=0.00567$ $\quad\delta=0.00610$
$(\gamma+\delta)\,D=0.00002174$ $\quad(\gamma+\delta)\,F=0.00001784$
$(\gamma+\delta)\,E=0.00001940$

GRATING No. 8.

$\gamma=0.014256$ $\quad\delta=0.003299$
$(\gamma+\delta)\,D=0.00002174$

GRATING No. 9.

$\gamma=0.013470$ $\quad\delta=0.006999$
$(\gamma+\delta)\,D=0.00002173$

GRATING No. 10.

$\gamma=0.002878$ $\quad\delta=0.022486$
$(\gamma+\delta)\,D=0.00002173$

A very minute change in the distance apart of the wires or in the openings produces a proportionally great change in the spectra; therefore a small inequality in the distance apart of the centres of the threads will cause a marked indistinctness

in the lines of the spectra. However great may be the accuracy of construction of the gratings, it has its limits; and this is the reason why even with fine gratings certain lines in some spectra cannot be seen with such clearness as will permit their position to be accurately observed. This was the case in grating No. 1 with line B^{II} and the lines in the fifth and following spectra; in No. 2, with the line C^{I} and some others; in No. 3, with B^{I}, B^{II}, etc. The lines B and H are the hardest to see in every spectrum and with every grating; because they lie nearly at the ends of the spectrum, and the intensity of their light is small in comparison with that of the other lines of the spectrum.

The grating with which the greatest number of spectra could be measured with accuracy was No. 4. For some spectra I have used the eye-piece prism in order to determine the position of such lines as were covered by other spectra; these are C^{III}, C^{IV}, G^{III}, H^{III}, which are not visible except by means of the prism. On account of this mutual overlapping only those lines can be seen in the spectra far away from the axis which are contained in the most intense portion; this is in the neighborhood of the line E. All the spectra observed with grating No. 4 have peculiar properties; namely, the spectra near E^{VI} and E^{VII} become weaker, E^{VIII} cannot be seen, but those that follow are again visible; yet in each of these last a different color seems to prevail. If we calculate the location of L^{I} for a single opening, making $\gamma = 0.000549$ — *i. e.*, the distance apart of the wires of the grating—we find that this falls almost where E^{VIII} should be. We will see later that this is also the explanation of the phenomenon described.

With grating No. 5 the fourth spectrum is almost three times as bright as the third. The explanation of this is to be sought in the fact that with this grating L^{I} falls in the third spectrum.

With grating No. 6 E^{VII} and E^{VIII} cannot be seen. At each of the E's that follow a different color prevails, *viz.*, by E^{IX}, blue; E^{X}, bright blue; E^{XI}, green; E^{XII}, yellow; and E^{XIII}, orange. For this grating L^{I} falls in the space where E^{VII} and E^{VIII} ought to be. The colors which are prominent in E^{IX}, etc., are, with reference to their places, those which would be seen in the second spectrum of the first class if the opening were 0.006759, which is the width of the openings of grating No. 6.

With grating No. 7 the third spectrum is only half as bright as the second.

Since with grating No. 8 the first spectrum occupies a space of about two minutes, the lines cannot be seen even if the spectrum is magnified fifty times. In the third and fourth spectra D was visible, but the other lines were not seen sharply enough to allow of measurement. The fifth spectrum is almost invisible; the sixth is weak, the seventh is much less bright than the sixth. With this grating one can distinguish most plainly the spectra of the first class. In order to calculate their positions and compare them with observations, the width of the single opening must be taken as δ instead of γ with this grating, in which the thickness of the wires is less than the width of the openings; and the same is true in all other such cases. The explanation will appear from special experiments which I have made on the question.

With grating No. 9, also, the lines could not be seen in the first spectrum. The third is almost invisible—there is scarcely a faint trace of its existence; similarly with the sixth and ninth spectra. In the spaces where these spectra should fall, L^{I}, L^{II}, etc., come; but in order to calculate their positions, δ must be taken in place of γ for the width of the opening.

With grating No. 10 the lines could first be clearly seen in the fourth spectrum. The eighth spectrum is less bright than the tenth, and the ninth seems to be absent; similarly the eighteenth is apparently invisible. For this grating, also, L^{I} and L^{II} fall in the spaces where the spectra are wanting.

From the close agreement of the values of $(\gamma+\delta)$ D, etc., with different gratings, one can judge of the degree of precision, which is not unimportant. If any one has doubt as to the possibility of such accuracy with reference to the quantities γ and δ, he need only realize that the method is to measure, say, 100 turns of the screw on which the threads are wound, by means of the microscope described above, and to divide the measured distance by the number of grooves taken; by which process $\gamma+\delta$ is often measured accurately to the sixth decimal place.

The following laws are deduced at once from the experiments with various gratings:

With two different gratings constructed of wires of uniform thickness and having a constant width of opening, the size of the

spectra which arise owing to the mutual action of a great number of beams diffracted through the narrow openings, and their distances from the axis, vary inversely as the distance between the centres of two openings, or, what is the same thing, as $\gamma+\delta$.

With perfect spectra of the second class the distances [from the axis] *of rays of the same color in different spectra form an arithmetical series whose common difference is equal to the first term.*

With a grating the thickness of whose wires and the width of whose openings are expressed in fractions of a Paris inch,

$$B=\frac{0.00002541}{\gamma+\delta},\quad C=\frac{0.00002425}{\gamma+\delta},\quad D=\frac{0.00002175}{\gamma+\delta},$$

$$E=\frac{0.00001943}{\gamma+\delta},\quad F=\frac{0.00001789}{\gamma+\delta},\quad G=\frac{0.00001585}{\gamma+\delta},$$

$$H=\frac{0.00001451}{\gamma+\delta}.$$

The relative spaces occupied by the different colors in any one spectrum from a grating are remarkable. For instance, the space CD is to the space GH nearly as 2 : 1; while in a spectrum produced by a flint-glass prism of only 27° the ratio of these spaces is almost 1 : 2, and with a prism of water nearly as 2 : 3.

I have already called attention to the fact that in order to see the lines in the spectra of the second class, the eye-piece of the telescope must be accurately focused in such a way that, if the grating were away, the vertical opening at the heliostat would be clearly seen. A small displacement of the eye-piece makes the lines indistinct or invisible. The rays, therefore, after they are modified by the grating, diverge from a point which is at the same distance from the grating as is the opening at the heliostat.

If a grating is placed at a considerable distance in front of the objective in such a manner that the rays falling upon the telescope from the heliostat must pass through the grating, the lines of the spectra appear, with the eye-piece in the position described above, just as they would if the grating were close to the objective; the distances of the colored beams from the axis

are, however, found to be smaller. The reason for this may be seen at once. [*Explanation omitted.*]

If light is allowed to pass through two identical gratings at the objective—*i.e.*, if two identical gratings, one behind the other, are placed in front of the telescope—the dimensions of the spectra are the same as they would be with one of the gratings. If two different gratings are used, the distances of the spectra from the axis are what they would be if the finer of the two gratings was placed by itself in front of the objective.

Mutual Action of Two, Three, Etc., Diffracted Beams.

If by means of two screens whose edges are straight and vertical the exposed surface of a grating is reduced to the interval of one opening, and if, when placed in front of the telescope, sunlight is allowed through this opening only, the spectra are the same, of course, as those obtained by means of a single narrow opening of the same width. The spectra are, therefore, of the first class, as represented by Fig. III., Table I. If the two screens before the grating are moved back so that light passes through two openings, and consequently two diffracted beams fall upon the objective, there is seen through the telescope a new sort of spectrum in the space which $L^I L^I$ occupied before, as is shown at M^I, M^{II}, etc., in Fig. II. I shall call these spectra "*imperfect spectra of the second class.*" With reference to the colors and their order of occurrence, this space $M^I M^I$ is similar to $L^I L^I$ in spectra of the first class; the space $M^I M^{II}$ is like $L^I L^{II}$, etc.; they are visible in that space only in which, with a single opening, $L^I L^I$ comes. Beyond this space the spectra are exactly like those given by a single narrow opening. Accordingly, if two diffracted beams fall upon the objective, one sees the imperfect spectra of the second class and the spectra of the first class at the same time. We shall see below what relation there is between the dimensions of these imperfect spectra of the second class and the width of the openings, etc. I shall designate the red ends of these spectra by M^I, M^{II}, M^{III}, etc.

If the two screens in front of the grating are so placed as to allow light to pass through three openings, the space $M^I M^I$, Fig. II., is divided into new spectra almost in the same manner

as the space L^IL^I was before. These new spectra are similar to the others in the order of their colors, and are present only in the space M^IM^I. I shall call these new spectra "*spectra of the third class,*" and shall designate their red ends by N^I, N^{II}, N^{III}, etc. Beyond the space M^IM^I the spectra of the second class appear almost exactly as they did when there were two diffracted beams. However, with certain gratings their distances from the axis were changed slightly. The spectra of the first class appear also as they did when there were two diffracted beams. There are seen, then, in this case three different kinds of spectra, *viz.*, third class, imperfect second class, and first class. The experiments described below will show what relation exists between the dimensions of the spectra of the third class and the width of the openings.

With four diffracted beams there are seen spectra of the first class, the imperfect second class, and the third class; but the last are noticeably smaller than they were when there were three diffracted beams. The spectra of the second class have changed only slightly.

With five diffracted beams the spectra of the third class are smaller than with four, while those of the second class have changed only a little. With six diffracted beams the spectra of the third class are smaller than with five; with seven beams, smaller than with six, etc., until they finally become so small that they can be distinguished no longer, and there is seen only a bright, colorless line which has the same appearance exactly as the opening at the heliostat would have if the grating were away. The spectra of the second class have in the mean time, owing to the increase in the number of diffracted beams, gradually changed both in their constitution and in their distances from the axis, and have approached more nearly in every respect the perfect spectra of the second class. The lines of the spectra become visible, however, and the light becomes homogeneous, only when a great number of equally diffracted beams at equal distances apart mutually influence each other.

In the following experiments, N^I, N^{II}, N^{III}, etc., are, as I said above, the red ends of each spectrum of the third class; just as M^I, M^{II}, etc., are the red ends of each imperfect spectrum of the second class. This method of writing symbols has also been used in the spectra of the first class. What I said there also

in regard to the accuracy of the angles holds true here. The angles M^{I}, M^{II}, etc., are always the distances from the axis. [*Experiments omitted.*] By careful consideration of the degree of accuracy of the above experiments, it follows, within the limits of error, that:

Using one and the same grating, but a different number of openings, the distances of the spectra of the third class from the axis and their dimensions vary inversely as the number of diffracted beams, i.e., as the number of grating openings, beginning with three.

With different gratings and with the same number of openings, the distances of the spectra of the third class from the axis and their dimensions vary inversely as $\gamma+\delta$.

For spectra of the third class, the distances from the axis form an arithmetical series whose common difference equals the first term.

Further:

FOR THREE BEAMS.

$$N^{I} = \frac{0.0000208}{3\,(\gamma+\delta)}, \quad N^{II} = 2\,\frac{0.0000208}{3\,(\gamma+\delta)}.$$

FOR FOUR BEAMS.

$$N^{I} = \frac{0.0000208}{4\,(\gamma+\delta)}, \quad N^{II} = 2\,\frac{0.0000208}{4\,(\gamma+\delta)}, \quad N^{III} = 3\,\frac{0.0000208}{4\,(\gamma+\delta)}.$$

FOR FIVE BEAMS.

$$N^{I} = \frac{0.0000208}{5\,(\gamma+\delta)}, \quad N^{II} = 2\,\frac{0.0000208}{5\,(\gamma+\delta)}, \quad N^{III} = 3\,\frac{0.0000208}{5\,(\gamma+\delta)}.$$

The space KM^{I}, Fig. II., Table I., which is seen when there is mutual action of two diffracted beams, contains the spectra of the third class if there are three beams, as I have stated above; therefore, with three, four, etc., beams, M^{I} can no longer be seen; and in these cases M^{II} is the red end of the first spectrum of the second class, a fact which must not be overlooked when comparing perfect and imperfect spectra. Since the spectra of the third class which are contained in the space KM^{I} become smaller as the number of beams which mutually influence each other increases, and since the spectra be-

come ultimately so small that they cannot be distinguished, thus producing in K only a bright, colorless line, the space between K and M^{I} must be without light; because, with an increase in the number of beams, the spectra of the second class do not change their position and magnitude, comparatively.

With grating No. 9, M^{IV} is invisible because it falls nearly at L^{I}. Since in this grating γ is greater than δ, the latter must be used instead of the former in calculating L^{I}. Why this is so will appear at once. In order to produce spectra of the first class, one must have two edges lying close together, by which the light is diffracted. It is not necessary that these two edges should face each other; they can also be turned reversed, provided only that they lie near enough, as is the case with a narrow metal strip, a thread, or a wire. In these cases, however, the spectra of the first class cannot be seen plainly, because they occur at the same places where non-diffracted white light comes. By means of a telescope, however, one can easily convince himself of their existence. For this purpose I stretched a wire 0.02287 inch thick across the middle of an opening in a screen, which was $\frac{1}{4}$ inch wide, and placed it in front of a telescope, with the thread vertical. The spectra of the first class which must arise, owing to the width of the opening in the screen—$\gamma = \frac{1}{4}$ inch—would be too small to notice, and the opening at the heliostat must appear through the telescope almost as if the screen were away; if, however, the thread stretched across the opening produces spectra, these must be seen on either side of the opening at the heliostat; this is actually the case. Owing to the width of the opening at the heliostat, it is seen so brightly that the eye can scarcely bear it; and on each side are seen spectra of the first class. On account of the intensity of light in the middle, L^{I} could not be measured; but the following two could be. I found $L^{II} = 6' 16''$ and $L^{III} = 9' 30''$. If we assume the above thickness of the wire as the value of γ, in order to calculate L^{II} and L^{III}, angles are obtained which agree as closely with those observed as could be expected under the circumstances.* In order, therefore, to produce spectra of the first class, it is not necessary that the edges which diffract the light should face each other; they

* These experiments deserve to be continued, because, in some cases, there are discrepancies which exceed the limits of probable error.

can be reversed, as is the case with a wire. With grating No. 9 the edges of the wires are nearer than those of the intervening spaces; therefore, the former produce the spectra of the first class.

The imperfect spectra of the second class are often modified by the position of spectra of the first class; and these modifications depend, besides, upon the increase or decrease in the number of the beams which mutually interfere. The exact law of these small changes cannot be deduced from the experiments so far made, and other experiments are therefore necessary. This much, however, can be seen from the above, that with different gratings the distances of the imperfect spectra of the second class, from the axis, and their dimensions, are nearly inversely as $\gamma+\delta$; further, that with two beams M^{I} is considerably smaller than the differences $M^{II}-M^{I}$, $M^{III}-M^{II}$, etc.; and therefore, with reference to the succession of the distances of the colored beams from the axis, these spectra are markedly different both from those which arise from a round opening and from those which come from a long, narrow opening. With gratings where δ is greater than γ, it is not difficult to deduce a law for the imperfect spectra of the second class—for instance, with gratings No. 6 and No. 10.

The angles L^{I}, L^{II}, etc., cannot be determined with great accuracy when there are two, three, etc., diffracted beams; therefore no certain conclusion can be drawn from the variations in these angles with three, four, etc., diffracted beams in the case of grating No. 10. One reason lies in the fact that it is almost impossible to make three or four intervening openings exactly equal, although the centres of the wires may be at equal distances from each other. We know from the experiments with one diffracted beam how great the variation in the distances of the spectra is, if, with an opening already small, the width changes even slightly; therefore with three diffracted beams the distances of the spectra of the first class from the axis may easily be found to be different from what they are with two beams, etc. This inequality [*of the spaces*] has much less influence upon spectra of the second and third class.

MEMOIRS ON

MUTUAL ACTION OF BEAMS DIFFRACTED IN WATER AND OTHER REFRACTING MEDIA.

[*Experiments omitted.*]

The conclusion is: *In different refracting media, with identical gratings, the sines of the angles of the diffracted beams vary inversely as the indices of refraction.*

MUTUAL ACTION OF BEAMS DIFFRACTED BY REFLECTION.

[*Experiments omitted.*]

MUTUAL ACTION OF BEAMS DIFFRACTED THROUGH ROUND AND QUADRANGULAR OPENINGS.

[*Experiments omitted.*]

When seen under a microscope the barbs of most birds' feathers contain regularly arranged fine openings. If one looks through the barbs at a brilliantly illuminated point which is not too near, he can see, even with the naked eye, colored spectra, which have a definite position. If such a feather is placed in front of a telescope and light coming from a round opening near a heliostat is allowed to pass through it, spectra of the first and second class are seen. The spectra which were seen with the naked eye were those of the first class, which are very large but of feeble intensity; and therefore, when seen through the telescope, which magnifies greatly, they may be easily overlooked, if attention is not paid to their distance from the axis.

With some gratings made of parallel wires, one sees through the telescope, beyond the space which is occupied by the length of the spectra—that is, in the dark portion of the field—what are apparently the wires of the grating itself, which is, however, impossible, if one considers the path of the light; one might perhaps believe that this light is due to interval reflections at the surfaces of the objective. But this is not the case; for the eye-piece can be pushed in or drawn out an inch, and the lines still remain visible. These lines have also a definite color; namely, one is always a reddish yellow, the second is blueish green, the third is again reddish yellow, etc. I shall return to this subject at another opportunity.

The phenomenon which is seen through a telescope when light is allowed through an opening of the shape of an equilateral triangle placed in front of the objective is also interesting.

It may be surprising that so many phenomena could up to this time have escaped the notice of investigators; and that, for instance, the simple law, that in diffraction through a single opening the inflection of the rays varies inversely as the width of the opening, was not only not found, but that results were obtained differing widely from this. The explanation lies in the mode of observation. One would fall into similar errors if, for instance, he tried to determine the path of colored light through glass lenses by intercepting and measuring this refracted light at different distances. This mode of observation is to blame for the fact that the phenomena of the mutual influence of beams of light have escaped the attention of investigators,* phenomena which must be known before the laws of diffraction can be learned. For if light coming through a grating is intercepted by a white surface or a piece of ground-glass, one cannot see, even on a small scale, what is observed through a telescope when used with the grating; and in general nothing is seen.

It is remarkable that the observed laws for the mutual action and diffraction of beams can be deduced from the principles of wave-motion; that, from a knowledge merely of the angle of diffraction of the light and of the distances at which the beams interfere, the magnitude of a vibration of light, for every color, can be expressed by means of an extremely simple equation, and that these determinations, under the most varying conditions, agree to a high degree of accuracy; further, that these same principles furnish an explanation of the cause of the origin of the lines and bands which are seen in a prismatic spectrum, etc. On another occasion I shall make known the theory of interference and diffraction of beams of light.

The phenomena of interference and diffraction of light are,

* T. Young had already observed that the colored fringes which are seen in the interior of the shadow of a hair vanish if one edge is covered; so that the beams of light going by both edges must combine to produce the interior color-bands.

as we see from the laws deduced, infinite in number; and all that was previously known amounts to only a few special cases. The theory will also make us acquainted with these phenomena which can be subjected to no new investigation following the path which I have taken.*

I cannot repeat too often that all apparatus to be used in these experiments must be perfect to a high degree; and one can judge of the degree of this perfection from the ratio of the dimensions of the grating to the dimensions of the spectra, etc. An inequality or imperfection, apparently unimportant, can produce a great indistinctness or even the entire vanishing of the phenomena; therefore, one must consider well everything that can be of injurious influence. More than with all other optical phenomena, one must be on one's guard in these experiments against optical illusions.

It will be reward enough for me if, by the publication of the present experiments, I have directed the attention of investigators to this subject, which still promises much for physical optics and appears to open a new field.

* Among these are the color-fringes which are seen in the shadow of a single edge of a body; also the phenomena which Herr Hofrath Meyer observed long ago and described in the *Göttinger Commentaren*, Vol. IV., p. 49.

Short Account of the Results of New Experiments on the Laws of Light, and their Theory.

Gilbert's *Annalen der Physik*, Band 74, p. 337–378. *Edinburgh Journal of Science*, VII., VIII., 1827, 1828.

CONTENTS

Short Account of the Results of New Experiments on the Laws of Light, and their Theory.

Read in the Mathematical-Physical Section of the Academy, June 14, 1823.

I PUBLISHED a year ago, in a memoir which is printed in the eighth volume of the *Denkschriften* of the Royal Bavarian Academy of Sciences, an account of some new experiments concerning *diffraction* of light and those phenomena which occur owing to the mutual action of diffracted beams of light; and the *laws* which can be deduced from these experiments were also developed. Guided by these deductions, I have continued these experiments since then; and what follows is an account of those of my results which are suited for communication in a brief description. I must, however, assume as known the larger portion of what the above-mentioned memoir contains on this subject.

[*Résumé of previous papers is omitted.*]

In this memoir I shall designate by B, C, . . . H *colored rays* of different kinds: B is a red ray which lies near the end of the spectrum; C is deeper in the red; D is orange-colored; E, green; F, blue; G, indigo; H, violet. In every spectrum of sunlight which consists of *perfectly homogeneous* light there are found in the places named fixed lines or streaks which are distinguished from the other countless lines of the spectrum either by their intensity or by their position.*

The angles from the axis through which the rays B, C, etc., are deflected by the grating I designate by B^{I}, C^{I}, etc., in the *first* spectrum, which is nearest the axis; by B^{II}, C^{II}, etc., in the *second;* by B^{III}, C^{III}, etc., in the *third* from the axis, etc. From the experiments which are described in detail in my

* [*A long note giving the conditions for securing a pure prismatic spectrum is omitted.*]

second memoir, it follows that, with all gratings, if γ denotes the width of a single grating space, and δ the width of a single wire, expressed in fractions of a Paris inch, the arcs of these angles are as follows, the radius being taken equal to 1:

$$B^{I}=\frac{0.00002541}{\gamma+\delta},\quad C^{I}=\frac{0.00002425}{\gamma+\delta},\quad D^{I}=\frac{0.00002175}{\gamma+\delta},$$

$$E^{I}=\frac{0.00001943}{\gamma+\delta},\quad F^{I}=\frac{0.00001789}{\gamma+\delta},\quad G^{I}=\frac{0.00001585}{\gamma+\delta},$$

$$H^{I}=\frac{0.00001451}{\gamma+\delta}.$$

Further, that

$$B^{II}=2\ B^{I},\ C^{II}=2\ C^{I},\ \text{etc.}$$
$$B^{III}=3\ B^{I},\ C^{III}=3\ C^{I},\ \text{etc.}$$
$$\text{Etc.}$$

The numerator in these general expressions is a number which is absolutely constant for each definite colored ray, but different for different rays, however varied the cases are, and which refers, as is easily seen, to a definite absolute scale; in this case to the Paris inch. If for each colored ray this number is designated by ω, and if the angle of deflection of one and the same ray in the first spectrum is designated by ϑ^{I}, in the second by ϑ^{II}, in the third by ϑ^{III}, etc., then,

$$\vartheta^{I}=\frac{\omega}{\gamma+\delta};\quad \vartheta^{II}=\frac{2\,\omega}{\gamma+\delta},\ \text{etc.}$$

And if, further, ν denotes the number which gives the order of the spectrum (for the axis $\nu=0$; for the first spectrum $\nu=1$; for the second, $\nu=2$, etc.; and it can never be a fraction); and if, for the sake of brevity, we call the sum of the width of one grating opening and of one line of the grating, or $\gamma+\delta$, equal to ε, we have as a general formula

$$\text{(I.)}\qquad \vartheta^{(\nu)}=\frac{\nu\omega}{\varepsilon}.$$

According to the results of the previous experiments (and as is shown by the general formula (I.) which is derived from them), the angles of deflection of the same colored beams in

the series of spectra formed by the grating are in the ratio of the numbers 0, 1, 2, 3, etc. The experiments from which these results are deduced gave, however, such small angles that for them the sine, tangent, and arc do not sensibly differ. With my finest grating, where $\epsilon = 0.001952$ inch, D^{I} was equal to 38′ 19″.3. If the angles were larger, that is, if the gratings were many times finer, one might think it probable, from certain considerations, that the arcs themselves do not have the relations given above, but that some trigonometrical function of them does. In accordance with the theory which will be discussed later, the *sines* of the angles should have the given relation if the light falls normally upon the grating. Partly in order to verify this directly by experiment, partly because the laws of this modification of light can be deduced with more accuracy from larger spectra, and conclusions as to the theory of these phenomena can be drawn with more certainty, it was greatly to be desired, it seemed to me, that gratings should be made much finer than those which I used in my earlier experiments. The distances apart of the lines or wires of such fine gratings must, however, be exactly equal to such a high degree that the fixed lines of the spectra can be seen; otherwise the distances of the colors from the axis cannot be measured.

It will not be considered possible by any one who is acquainted with the difficulties of the work to prepare a screw much finer than that which I made for my earlier experiments. With a special apparatus I succeeded in scratching parallel lines upon a piece of plane glass, covered with gold-foil, which were at such distances from each other that ϵ was equal to 0.00114 inch. If one tries to scratch lines closer together than this, no gold is left on the glass, and there are, consequently, no apertures. The spectra produced by such a grating, where $\epsilon = 0.00114$, are much larger than those obtained before, and the fixed lines were plainly seen in them; but the results obtained were still far from being able to decide the question at issue.

It is entirely immaterial in a grating which is to be used for these experiments whether the threads out of which it consists are opaque, translucent, or transparent. A grating of glass fibres, for instance, produces these phenomena as well as one made out of metal wires. I covered, therefore, one side of a piece of plane glass with a layer of grease so thin that it

could scarcely be recognized by the eye alone; in this grease I scratched parallel lines which were only half as far apart as the finest of those ruled on the gold-foil. With this grating I obtained spectra in which the fixed lines could be plainly seen, and which were therefore suited for measurements of the distances from the axis. In no layer of grease or varnish is it possible to rule finer parallel lines than these, which shall be at equal intervals.

It was only by means of the diamond that I succeeded in producing finer gratings. A machine specially made for the purpose enabled me, by using a diamond-point, to rule lines in the surface of a piece of plane glass which were almost perfectly parallel. If one is so fortunate as to find a good diamond-point, lines can be ruled with this machine which are so close together that they cannot be distinguished by the strongest compound microscope. It is, however, not enough to be able to rule in a given space a great number of lines which shall still be separate from each other; these lines must be ruled so accurately at equal distances apart that the variation shall not amount at the most to the one-hundredth of this distance. By means of my machine I have obtained a grating in which $\epsilon = 0.0001223$ inch, and whose lines are so evenly spaced that the fixed lines of the first and second spectra obtained with it can be plainly seen.*

* By means of my machine one can rule parallel lines which are separated by spaces as wide as themselves, and which are so close together that 3200 of them lie in a Paris inch; but I have not yet succeeded in making their distances from each other so exactly equal that in this distance, which is 0.00003125 inch, errors of one part in a hundred—*i.e.*, of 0.00000031—are not present; and perhaps this would not be possible for the hand of man, whatever machine is used. Since no use can be made of 100 or 200 parallel lines, and since with such fine gratings there must be several thousand lines in order to produce intense perfect spectra, it is evident that it was largely a question of fortune, with $\epsilon = 0.0001223$, to find a diamond-point which could rule several thousand such fine lines without changing. So far, I have succeeded in making only one grating as fine as this. If the diamond-point changes during the ruling, the previous work is useless. Without one's intending it, the point often makes stronger or weaker lines. Even with the most powerful microscope one cannot be sure whether the point is suited to rule proper lines. A diamond which appears less pointed than another often rules the finer lines; and therefore a suitable point can be found by experiment only. What renders the matter more difficult is that a slight change in the inclination or in the position of the diamond,

By means of this grating spectra are obtained which are as long as those obtained from large prisms; and even in the first spectrum the D line (in the orange) can be seen practically double, so that the space between can be measured. And since with this grating D^{I}, for instance, equals 10° 14′, the law of modification of the light produced by it can be deduced with great accuracy.

When the light fell normally upon the grating in which $\varepsilon =$ 0.0001223, I obtained

$C^{I} = 11° \; 25' \; 20''$	$F^{I} = 8° \; 26' \; 6''$
$C^{II} = 23° \; 19' \; 42''$	$F^{II} = 17° \; 3' \; 34''$
$D^{I} = 10° \; 14' \; 31''$	$G^{I} = 7° \; 27' \; 19''$
$D^{II} = 20° \; 49' \; 44''$	$G^{II} = 15° \; 3' \; 9''$
$E^{I} = 9° \; 9'$	$H^{I} = 6° \; 52' \; 36''$
$E^{II} = 18° \; 32' \; 34''$	

The angles are so large that the arcs, sines, and tangents are sensibly different. Since the instrument with which the angles are measured gives, without repeating the observation, readings accurate to 4″, one can easily decide how trustworthy the readings are.

The third spectrum, the fourth, and the following were seen satisfactorily in these experiments with this grating; but the fixed lines in the various colors could not be recognized with sufficient precision to have their distances from the axis measured as accurately as in the first and second.*

with reference to the plane of the glass often seriously influences the strength of the lines. Since every line must be drawn separately and with the greatest care, one can easily understand how much time and patience is required in order to rule a few thousand lines with sufficient accuracy.

* However great the accuracy of the ruling with reference to the distances apart of the lines of the grating, it naturally has its limits, and perfect accuracy cannot be expected; a statement which is true of everything made by the hand of man. The amount of the displacement or blurring of the fixed lines of the different spectra which would be produced by a small inequality in the intervals of the parallel lines of a grating, can be deduced from the equation (I.) $\vartheta^{(\nu)} = \frac{\nu\omega}{\varepsilon}$. For it follows from it that $d\vartheta^{(\nu)} = - d\varepsilon \frac{\nu\omega}{\varepsilon^2}$; consequently an equality which amounts to only one-hundredth of the very small distance ε (*i.e.*, $d\varepsilon = 0.00000122$ inches) will pro-

The measurements given for the first and second spectras are sufficient, however, to allow one to decide upon the law of this modification.

Since the distance between the parallel lines, ruled on the *glass grating,* as I shall call it, must be known most accurately, and since these lines are seen with great difficulty even with the strongest microscope, and in any case cannot be counted, I try to rule the first and last line of the grating somewhat stronger than the others, and I measure their distance apart by means of a microscopic apparatus prepared for the purpose. The ruling-machine counts the lines as they are ruled; and so I know how many there are in the ruled space; *e.g.*, in the grating just discussed there are 3601 lines. Therefore, from a knowledge of the distance of the first line from the last, the distance between the centres of two—that is, ϵ—can be determined to a high degree of accuracy. The ratio, too, of the thickness of a scratch to the width of the opening between two lines can be very closely determined. If the lines were so wide that one touched the other, and consequently there was no aperture between, no light could be regularly reflected from the ruled surface; it would be scattered as from a ground surface. If the apertures were as wide as the scratches, the ruled surface could reflect only half as much light as an unruled surface of the same size. The ratio, therefore, of the light regularly reflected from the ruled glass surface to that which is reflected by an unruled glass surface of the same size is the same as that of the width of the aperture between two scratches to the width of the lines [*i.e., to* ϵ].* It need scarcely be noted that in the experiments made in this

duce a displacement of the fixed line D, in the first spectrum, of 6′ 5″; in the second, of 12′ 10″; in the third, of 18′ 15″, etc. This is the reason why the fixed lines may be well defined in the first and second spectra, and yet not be so in the third, fourth, etc. Whoever has to do with small magnitudes, not merely in numbers, but in experiments, will know how to estimate this accuracy, and will have an idea as to the difficulty with which it is obtained.

* Even if the amount of the reflected light could be accurately measured, this conclusion must be regarded as only approximately true for reasons to be discussed later. With gratings for which ϵ is still smaller than this it would be less true. In the experiments described in this paper the determination of this ratio is of no importance whatever; the magnitude of ϵ is alone of interest.

way care must be taken to have no light reflected from the second surface of the glass.

Using another grating, where ε was equal to 0.0005919 inch, I obtained, with light at normal incidence,

$C^{I} = 2° 20' 57''$	$E^{V} = 9° 28' 3''$
$D^{I} = 2° 6' 30''$	$F^{I} = 1° 44' 19''$
$D^{II} = 4° 13' 7''$	$F^{II} = 3° 28' 45''$
$D^{III} = 6° 20' 7''$	$F^{III} = 5° 13' 23''$
$D^{IV} = 8° 27' 43''$	$F^{IV} = 6° 58' 18''$
$D^{V} = 10° 35' 53''$	$G^{I} = 1° 32' 22''$
$E^{I} = 1° 53' 7''$	$G^{II} = 3° 4' 57''$
$E^{II} = 3° 46' 17''$	$G^{III} = 4° 37' 30''$
$E^{III} = 5° 39' 50''$	$H^{I} = 1° 25'$
$E^{IV} = 7° 33' 41''$	$H^{II} = 2° 50' 11''$

All the observations, with both glass gratings, can be expressed very approximately by the equation

$$\text{(II.)} \quad \sin \vartheta(\gamma) = \frac{\nu\omega}{\varepsilon}.$$

If the light is incident *normally*, therefore, the *sines* of the distances of a colored ray from the axis in the different consecutive spectra are in the ratio of the numbers 0, 1, 2, 3, 4, etc.*

The glass grating, $\varepsilon = 0.0005919$, has the remarkable property that its spectra on one side the axis are more than twice as intense as those on the other. The scratches of this grating are visible under a microscope, but it is impossible to distinguish

* If it were not the sines, but the angles, of deflection of a colored ray in the different spectra which had the given ratio, then with the finer grating, where $D^{I} = 10° 14' 31''$, D^{II} should equal $20° 29' 2''$; according to experiment, however, $D^{II} = 20° 49' 44''$—that is, is 20 minutes greater. Consequently, the sines of the angles have the given relation. Calculation leads to a difference in the seconds, which, although small, is larger than can be ascribed to an error of observation. Whether this difference is to be ascribed to a slight imperfection in the grating, or whether it lies in the nature of the phenomena, can be decided only by a series of experiments with different fine gratings. I give in the table the angles exactly as they were observed, not allowing myself any correction whatever. I determined these angles very often, and always by repeating six times; yet I obtained angles for the bright colors which agree nearly always to within one second; notwithstanding which, small constant errors could cause the differences in the seconds which are mentioned above.

in them any special form. I sought the explanation in the idea that, on ruling the lines, the diamond-point had such a position with reference to the plane glass that one edge of each scratch was sharp, the other not so well defined; and I think that this hypothesis is confirmed by the following experiment: On a piece of plane glass, over which there was a thin layer of grease, I ruled parallel lines in such a way that with every line the grease on one side was sharply defined, on the other less so; and I obtained phenomena, in fact, through this grating which were like those seen through the glass grating mentioned.

If the incident light does not fall perpendicularly upon the grating, but is in the plane perpendicular to the lines of the grating, and *inclined* to the normal to the grating, the action is as if for this incident light the distance apart of the parallel lines—that is, ε—was smaller than when the incidence was normal, in the ratio of the radius to the cosine of the angle of incidence. The distances of the spectra from the axis (ϑ) must therefore become greater as the angle of incidence increases; because (as equation (II.) shows) the sines of this distance increase in the same ratio as ε decreases. If we designate, therefore, by σ the *angle of incidence—i.e.*, the angle between the incident beam and the normal to the grating—we might think we could safely conclude from equation (II.)—*i.e.*, $\sin \vartheta^{(\nu)} = \frac{\nu\omega}{\varepsilon}$—that the following is true: $\sin (\vartheta)^{\nu} = \frac{\nu\omega}{\varepsilon . \cos \sigma}$. But from the theory of these phenomena, which will be discussed later, it may be predicted that in this case the spectra on the two sides of the axis will be no longer symmetrical; that therefore, *e.g.*, D^{I} must be larger on one side of the axis than upon the other. This is confirmed by experiment, as appears below. With gratings in which ε is not very small, the difference is not marked, it is true;* it is, however, exceptionally large with the grating in which $\varepsilon = 0.0001223$ inch. For instance, with this, for $\sigma = 55°$, D^{I} on the one side of the axis equals 15° 6′, and on the other, $D^{I} = 30° \ 33'$.

* In fact, with coarse gratings, if σ is not very large, one can use the expression $\sin \vartheta^{(\nu)} = \frac{\nu\omega}{\varepsilon . \cos \sigma}$ with sufficient accuracy, as I did in my memoir *A New Modification*, etc. We shall, however, find below an exact expression which is equally simple.

If in the theory of spectra of the first class the non-symmetrical ones, which consist of non-homogeneous light, and which contain no fixed lines,* are of importance, the *non-symmetrical spectra of the second class* are of immensely greater theoretical interest, because they show the fixed lines; and therefore the laws of this modification of light can be exactly deduced, and the theory of these phenomena can be most closely verified.† I give here the results of numerous experiments on the colored rays D and F. In order to indicate on which side of the axis ϑ lies—the angle measuring the distance of the colored ray from the axis—I mark with $-$I, $-$II, . . ., those spectra which lie upon that side of the axis on which the incident light is inclined towards the face of the grating; and with $+$I, $+$II, . . ., the spectra on the opposite side, where the inclined incident beam makes obtuse angles with the grating, so that D^{-I}, D^{+I} or D^{-II}, D^{+II}, . . ., mean the opposite positions of D in the first or second spectrum, etc. By σ is to be understood the angle of incidence in the sense described above—*i.e.*, the angle included between the incident ray and the normal to the grating.

EXPERIMENTS WITH THE GLASS GRATING $\varepsilon = 0.0001223$.

$\sigma = 55°$	
$D^{-I} = 15°\ 6'\ 36''$	$F^{-I} = 12°\ 44'\ 40''$
$D^{+I} = 30°\ 33'\ 10''$	$F^{+I} = 19°\ 58'\ 54''$
$D^{-II} = 27°\ 23'\ 18''$	$F^{-II} = 23°\ 16'\ 50''$
$\sigma = 50°$	
$D^{-I} = 13°\ 58'\ 12''$	$F^{-I} = 11°\ 43'\ 53''$
$D^{+I} = 20°\ 42'\ 51''$	$F^{+I} = 15°\ 53'\ 10''$
$D^{-II} = 25°\ 46'\ 20''$	$F^{-II} = 21°\ 47'\ 36''$
$\sigma = 45°$	
$D^{-I} = 13°\ 2'\ 37''$	$F^{-I} = 10°\ 54'\ 55''$
$D^{+I} = 17°\ 14'\ 14''$	$F^{+I} = 13°\ 37'\ 38''$
$D^{-II} = 24°\ 25'\ 30''$	$F^{-II} = 20°\ 33'\ 39''$

* *New Modification of Light*, etc.

† A simple hypothesis can be easily devised to satisfy all simple laws. For instance, the laws of refraction and reflection can be deduced from a great number of hypotheses. It is, therefore, good fortune if a new law is found which is apparently very complicated, because it has at least the advantage of limiting more closely the field of the hypotheses concerning light.

$\sigma = 40°$

$D^{-I} = 12° \; 17' \; 34''$	$F^{-I} = 10° \; 15' \; 29''$
$D^{+I} = 15° \; 8' \; 52''$	$F^{+I} = 12° \; 8' \; 12''$
$D^{-II} = 23° \; 18' \; 54''$	$F^{-II} = 19° \; 32' \; 57''$

$\sigma = 30°$

$D^{-I} = 11° \; 13' \; 22''$	$F^{-I} = 9° \; 18' \; 36''$
$D^{+I} = 12° \; 40' \; 30''$	$F^{+I} = 10° \; 17' \; 34''$
$D^{-II} = 21° \; 42' \; 5''$	$F^{-II} = 18° \; 4' \; 35''$
$D^{+II} = 28° \; 50' \; 5''$	$F^{+II} = 22° \; 30' \; 10''$

$\sigma = 20°$

$D^{-I} = 10° \; 33' \; 2''$	$F^{-I} = 8° \; 44' \; 10''$
$D^{+I} = 11° \; 19' \; 23''$	$F^{+I} = 9° \; 15' \; 22''$
$D^{-II} = 20° \; 46' \; 54''$	$F^{-II} = 17° \; 12' \; 45''$
$D^{+II} = 24° \; 14' \; 30''$	$F^{+II} = 19° \; 27'$

In these experiments, when measuring the first angle, the instrument could not be used in such a way as to allow repetition. These first angles are, therefore, in the seconds, somewhat less exact than when the light has normal incidence. Since, however, the differences between the two sides of the axis amount to several degrees, a few seconds do not affect the matter.

In all the experiments of this kind D^{-I} and D^{+I} are different, as is seen; similarly with F^{-I} and F^{+I}; and the same was true of the other colored rays, for which I have not given the observations. If the light is not incident normally, therefore, the spectra produced by the grating are no longer symmetrical on the two sides of the axis; and the difference in their positions is so marked when the angle of incidence is great that it could be detected even if it were only one-hundredth part as much. It would be, however, extremely difficult for the most clear-sighted physicist to deduce a *law* for these phenomena directly from the experimental results.

Let τ denote the angle made with the plane of the grating by a colored beam after diffraction (with normally incident light, therefore the complement of ϑ), and y a straight line drawn perpendicular to the plane of the grating from the micrometer-threads of the observing-telescope.* All of my observations can

* If a is the distance from the grating to the micrometer-thread (and therefore to the place where the image of the phenomenon is formed), $y = a \sin \tau$. [*a is measured along the beam of light.*]

then be expressed, with the accuracy obtainable in experiments of this kind, by the following formula:

$$\text{(III.)} \tan\tau(\pm\nu) = \frac{\sqrt{[\epsilon^2-(\epsilon.\sin\sigma\pm\nu\omega)^2].[4y^2+\epsilon^2-(\epsilon.\sin\sigma\pm\nu\omega)^2]}}{2y\,(\epsilon.\sin\sigma\pm\nu\omega)}$$

I have deduced this equation, without any approximation, from the principles of *Interference* which were proposed in 1802 by Dr. Thomas Young, and afterwards fully justified by the painstaking labors of Arago and Fresnel. In this formula ω denotes the *length of a light-wave.* Although this quantity is extremely small, we can deduce it with a high degree of accuracy from the experiments which are described in my memoir, *New Modification of Light*, etc.; and the results of which for the different colored rays are given in general formulæ on page 30. From the experiments with glass gratings we learn this quantity so exactly that, for the bright colors, hardly one-thousandth portion of ω can be uncertain. From the experiments with the finer gratings we obtain, by means of the angles for the first spectrum with normal incidence of the light, if $(C\omega)$ denotes the length of a light-wave for the ray C, $(D\omega)$ for the ray D, etc.,

$$\left.\begin{aligned}(C\omega) &= 0.00002422\\ (D\omega) &= 0.00002175\\ (E\omega) &= 0.00001945\\ (F\omega) &= 0.00001794\\ (G\omega) &= 0.00001587\\ (H\omega) &= 0.00001464\end{aligned}\right\}\text{ in fractions of a Paris inch.*}$$

[*Reduced to centimetres, this gives for* D *the wave-length* 0.00005888 *cm.* 1 *Paris inch*=2.70700 *cm.*]

* The fixed line B, near the red end, could not be seen well enough, owing to the great width of the image, to allow its position to be determined accurately. I shall try to make a greater number of fine gratings, in order to determine more accurately, where it is possible, the value of ω for the different colored rays; yet this value is now known sufficiently closely to decide whether the theory is confirmed by the experiments. If the values of $(D\omega)$, $(E\omega)$, etc., are deduced from the experiments with the coarse glass gratings, they are found to be somewhat greater in the eighth decimal place than those given by the finer gratings; this small difference simply points to a less exact determination of ϵ, which is more accurately known

It will strike one from equation (III.) that the value of τ depends somewhat upon y—*i.e.*, upon the distance at which the image of this phenomenon is formed; that therefore, according to the principles of interference, this light thus modified does not proceed exactly in a straight line, but in a curved one. The equation of this curved line, developed without any approximation, is

$$\text{(IV.)} \quad x^2[4\epsilon^2 - 4(\epsilon.\sin\sigma \pm \nu\omega)^2] = 4y^2(\epsilon.\sin\sigma \pm \nu\omega)^2 + [\epsilon^2 - (\epsilon.\sin\sigma \pm \nu\omega)^2](\epsilon.\sin\sigma \pm \nu\omega).*$$

Since these phenomena can be observed only with a telescope, which must have great power if the distances are to be measured with some accuracy, and therefore cannot be very short, y is always large in comparison with ω and ϵ. With my instrument, the distance of the micrometer-threads from the grating is 21.43 inches. It would be only when y is not very large in comparison with ω that the value of τ could vary with that of y; but if in equation (III.) y is put one-half as great as it actually was in my experiments, and then twice or three times as great, the same value of τ is always obtained.† As the equation shows, the value of $\epsilon^2 - (\epsilon.\sin\sigma \pm \nu\omega)^2$ vanishes, then, in com-

with the finer gratings. I do not need to recall the fact that the accuracy of the determination of the value of ϵ, however great it is by the help of the means which I use, has its limits. I hope to increase it still more in the future.

* Equations (III.) and (IV.) are developed for the case when the incident rays may be regarded as parallel. If the distance of the source of light were not great in comparison with ϵ, instead of $\epsilon.\sin\sigma$ in these equations one would have to write $\frac{\epsilon.\sin(\sigma+\frac{1}{2}\beta)}{\cos\frac{1}{2}\beta}$, where β is defined by the equation $\sin\beta = \frac{\epsilon.\cos\sigma}{a}$. In this last equation a denotes the distance of the source of light from the grating.

† I measured these angles also with a telescope of 4-inch focal length; but, as was to be expected from the equation, no other changes were found than those which must be ascribed to the diminished power; they were sometimes positive, sometimes negative. If a curvature of the modified light were noticeable, it might be in the case when the distance of the source of light is not so great; but in that case the experiments are subject to many difficulties, and demand a different apparatus, which must be perfect to a high degree.

parison with $4y^2$; and so it can be neglected. We therefore have, approximately,

$$\operatorname{tang} \tau^{(\pm\nu)} = \frac{\sqrt{[\varepsilon^2 - (\varepsilon . \sin\sigma \pm \nu\omega)^2]}}{\varepsilon . \sin\sigma \pm \nu\omega},$$

or (V.) $$\cos \tau^{(\pm\nu)} = \frac{\varepsilon . \sin\sigma \pm \nu\omega}{\varepsilon}.$$

This equation represents the experiments on page 49, with unsymmetrical spectra, as closely as does equation (III.). In both equations the sign $+$ gives the position of the colored rays on one side of the axis in the different spectra; the $-$ sign, those on the opposite side. In making comparisons, it must not be forgotten that in the experiments the angular distances from the axis are measured, but that τ expresses the angles measured from the plane of the grating. It is scarcely necessary to remark that in the special cases when, for instance, the above equation is used for the ray C, instead of ω, $(C\omega)$ is to be introduced; for the ray D, $(D\omega)$, etc. In these cases I designate for the ray C the corresponding τ by $(C\tau)$; that for D, by $(D\tau)$, etc. Equation (V.) becomes, accordingly, in these cases,

$$\cos (C\tau)^{(\pm\nu)} = \frac{\varepsilon . \sin\sigma \pm \nu(C\omega)}{\varepsilon};$$

$$\cos (D\tau)^{(\pm\nu)} = \frac{\varepsilon . \sin\sigma \pm \nu(D\omega)}{\varepsilon}, \text{ etc.}$$

If the incidence of the rays upon the grating is normal, $\sin \sigma = 0$, and equation (V.) becomes

$$\cos \tau^{(\pm\nu)} = \frac{\pm \nu\omega}{\varepsilon}.$$

Since in this case $\cos\tau = \sin\vartheta$, this is the equation (II.) deduced directly from the experiments on page 47; therefore those experiments also confirm the theory.

In any *other medium* than air I denote the index of refraction, *e.g.*, of the ray C by (Cn), of the ray D by (Dn), etc. It follows at once from the experiments on light diffracted in different media* that in the above equation $(C\omega)$ is to be replaced by $\frac{(C\omega)}{(Cn)}$; $D\omega$, by $\frac{(D\omega)}{(Dn)}$, etc.

* *New Modification of Light*, etc., p. 36.

If the unruled surface of a glass grating is covered with *black resin varnish*, which has nearly the same refracting power as the glass, then, as is well known, no light will be reflected from this side into the interior of the glass, and only the ruled surface of the glass reflects light. If light from an apparently very narrow opening in the window-shutter is incident upon the grating, and if, in the manner described, the light *reflected from the ruled surface* reaches the objective of the telescope, the phenomena are exactly the same as if the light, at the same inclination, had been transmitted *through* the grating—*viz.*, unsymmetrical spectra of the *second* class are seen. The intensity of these spectra is also great enough to allow the distances of the different fixed lines to be determined with considerable accuracy. I have made a great many experiments on these spectra arising from *reflection* at different angles of incidence, the details of which I pass over. Equation (V.) represents all the results as satisfactorily as was to be expected. By the development, from the principles of interference, of the formula for the phenomena due to reflection, the same expression is obtained as for transmitted light—*viz.*, without approximations, equation (III.). In this way, also, the theory is confirmed by experiment.

It is most remarkable that for a certain angle of incidence a part of one spectrum arising from reflection consists of *completely polarized light.* This angle of incidence is very different for the different spectra, and, still more notably, even for the different colors of one and the same spectrum. With the glass grating $\epsilon = 0.0001223$, $(E_T)^{+I}$ is polarized—*i. e.*, the *green* part of the first spectrum, if $\sigma = 49°$; $(E_T)^{(+II)}$, or the green part of the second spectrum on the same side, if $\sigma = 40°$; finally, $(E_T)^{(-I)}$, or the green part of the first spectrum on the opposite side of the axis, if $\sigma = 69°$. If $(E_T)^{(+I)}$ is completely polarized, the other colors of this spectrum are not completely so. With $(E_T)^{(+II)}$ this is less the case; and σ can be sensibly changed while this color remains polarized. For no angle of incidence is $(E_T)^{-I}$ as completely polarized as is $(E_T)^{(+I)}$. With a grating in which ϵ is greater than in this one, the angles of incidence must be entirely different if the above-mentioned spectra are to be polarized.*

* It would be premature to wish to deduce the law for this phenomenon from a small number of observations. This can be done with certainty

It is seen from equation (V.) that, if $\epsilon < (\epsilon . \sin\sigma \pm \nu\omega)$, $\cos\tau^{(\pm\nu)} > 1$, which is impossible. With normal incidence ϵ must be greater than $\nu\omega$, if $\tau^{(\nu)}$ is to be visible—*i.e.*, possible. If ϵ is less than ω, *no colored beam is any longer visible,* however the light is incident; and there remains only the white light in the axis—*viz.*, $\cos\tau^{(0)} = \sin\sigma$. This holds true for both transmitted and reflected beams, because the equations are the same for both. If $\epsilon = \omega$, $\tau^{(1)}$ would be 90°. If, therefore, a grating were made in which the distance apart of the centres of two of the parallel lines is smaller than ω, no spectrum at all would be seen, only the white beam at the axis.

We cannot believe that the polishing, which is done artificially upon glass, etc., is mathematically perfect. If the polish consists of unevennesses, which, in reference to their distances apart, are smaller than ω, they are of no detriment to either transmitted or reflected light, and no colors of any kind could be produced by them. Further, there would be no means of making these inequalities visible.* If the small inequalities had an action on the light—*e.g.*, by reflection according to the known laws—the rays would be scattered most irregularly, because the radii of curvature of these small unevennesses must be very small, and any regular reflection would be impossible. If a reflecting surface consists of unevennesses whose distances apart are smaller than ω, no spectrum is possible, as has been said; and light can be reflected only along the axis. For this ray $\nu = 0$; in which case equation (V.) gives the ordinary *law of reflection*—*viz.*, $\cos\tau^{(0)} = \sin\sigma$. This law follows, therefore, from interference, without the need of as-

only from a large number of gratings with which ϵ is widely different. Since in these experiments it is not necessary for the fixed lines of the spectra to be plainly seen, gratings can be made in which ϵ is sensibly smaller than in the finest ones which I have used up to this time. It is not improbable that the principles of interference will lead perhaps to a theory of the polarization of light. This is neither the place nor the time (at present) for me to communicate my opinions on this matter. Fortunately, other experiments of another kind are possible, which seem to have bearing upon this question; yet they are of a most delicate nature, as are the greater part of all the experiments on this subject.

* We can conclude from this what it is possible for one to see through a microscope. A microscopic object—*e. g.*, one whose diameter is equal to ω, and which consists of two portions—cannot be recognized as consisting of two parts. This shows us the limits of the power of a microscope.

suming a repelling force perpendicular to the reflecting plane.* The fact that at greater angles of incidence more light is reflected than at smaller ones follows just as simply and is in accord with experiment. It is remarkable that according to the exact formula (III.), at distances from the reflecting surface which are not great in comparison with ω—*i.e.*, at very short distances—the angle of reflection can be markedly different from the angle of incidence. One can easily see by an accurate consideration of this equation that at distances which permit one to make exact observations this difference is too small to be thought of; therefore, the ordinary experiments for the determination of the law of reflection can prove nothing against its deduction from the theory of interference.

It is seen, as a result of all the experiments with the different gratings, that the distances of the spectra from the axis increase as the grating-space—that is, ε—becomes smaller; and that, if the spectra are to be homogeneous, this ε must remain constant throughout the whole grating. If these spaces are unequal, the larger ε's will produce small spectra, the smaller ε's large ones, which mingle with each according to the degree of the irregularity. If the irregularity is very great, not even heterogeneous colors will be seen; and the light must be white in the whole region, as experiment shows is the case. It is, nevertheless, interesting to know what phenomena would arise if the grating-spaces were *unequal in a regular manner*—*i.e.*, if the inequality of the distances, whatever it is, is repeated at regular intervals. To this end I covered several pieces of plane glass with gold-foil, and upon them I ruled parallel lines whose distances apart were unequal, but unequal in various regular ways. I can give here in brief only a few of the results of these experiments, which must, besides, be continued further. The spectra which are seen with a telescope through this kind of a grating consist of homogeneous light; and their fixed lines are recognized most plainly, so that their distances from the axis can be accurately measured. If the grating-spaces of a grating with periodic inequalities are denoted by ε', ε'', etc., and if one of the identical portions, each of which

* [*A long note follows in which Fraunhofer upholds the wave-theory against its rivals. He says it accounts for regular refraction, the colors of thin plates and of mother-of-pearl, reflection from a rough surface, etc.*]

contains unequal ε's, is expressed by $\varepsilon'+\varepsilon''+\varepsilon'''+\ \ldots\ +\varepsilon^{n}$, then the results of the experiments are, that with normal incidence the distances of the different spectra from the axis are expressed by the following equation:

$$\sin \vartheta^{(\nu)}=\frac{\nu\omega}{\varepsilon'+\varepsilon''+\varepsilon'''+\ \ldots\ +\varepsilon^{n}}.$$

In whatever manner the ε's follow each other in the portion which contains the unequal parts and which is represented by the division of the equation, even if some of them are equal, the equation remains always the same, provided only that these portions cannot be themselves subdivided into smaller identical portions in which the ε's succeed each other in the same manner and same sense. This phenomenon, arising from gratings with periodic errors, is remarkable on account of the relative intensities of the different spectra, concerning which, however, no general statement can be made in a few words and without a figure. With some gratings of this kind, several spectra, or parts of them, may be entirely wanting, or may have such a feeble intensity that they can be observed with difficulty, while the following ones again are very intense. This has the great advantage, that in this case the fixed lines of those spectra can be observed which cannot be seen with gratings whose grating-spaces, ε, are equal, because they are covered by the neighboring spectra. For instance, with no grating having equal ε's can C^{XII} and F^{XII} be seen; but with a grating which has periodic errors, such that each portion consists of three ε's differing from each other in the ratio 25 : 33 : 42, C^{XII}, D^{XII}, E^{XII}, F^{XII} may be seen so plainly that their distances from the axis can be safely measured. For with such a grating the tenth and eleventh spectra are almost entirely absent. With this grating I saw even E^{XXIV}, and saw it so plainly that its distance from the axis could be measured. The relation between the intensities of the different spectra depends upon the relation between the ε's which follow each other in any one portion, and this is in many cases very complicated.* Similar gratings with peri-

* One can conclude from the experiments mentioned, as well as from others, that many of these phenomena are so complicated—at least, in all likelihood—that they cannot well be described in a few words; and I am for this reason forced to leave untouched much that might be of interest. The further one goes in these experiments, so much the wider becomes the

odic inequalities are obtained if two different gratings—*i.e.*, two in each of which ϵ is constant, but larger in one than in the other—are placed with their ruled faces together so that their lines are exactly parallel.

[*A page concerning gratings with concentric circular rulings is here omitted.*]

Addendum Concerning the Spectra of Flames, Moon and Star Light, and the Electric Light.

As is well known, the prismatic *color-spectrum* of the light coming from a *flame* (*lamplight*) does not show the dark fixed lines which are present in the spectrum of sunlight; instead of them there is in the orange a bright line which is prominent above the rest of the spectrum, is double, and is at the same place where in sunlight the double line D is found. The spectrum obtained from the light of a flame which is blown with a *blast-tube* contains several prominent bright lines. Of still greater interest for optical experiments is the fact that, by skilful blowing of the flame, the light of the *front half* of the flame can be dispersed no further by the prism, and, consequently, is simple homogeneous light. This light has, so far as I have investigated it, the same refrangibility as the D ray of sunlight. Simple homogeneous light which proceeds in all directions is, for known reasons, very difficult to produce, and can never be obtained with prisms directly; therefore this flame is of great use in many experiments.

By means of the large electrical machine of the physical cabinet of the Royal Academy, I obtained a spectrum of the *electric light*, in which I recognized a greater number of bright lines than I had seen before with weaker light. I determined the relative positions and intensities of the brightest of these.

The light of the *moon* gave me a spectrum which showed in

field which offers itself for new investigations. It is greatly to be regretted that they can be repeated so seldom by any one, owing to the fact that they demand very large, and, in part, expensive apparatus, and also a great deal of time. The fact that the sky must be most favorable makes one lose more time than would be believed, perhaps; which I feel all the more because the demands of business leave me only a few definite days in the month which are free for these investigations.

the brightest colors the same fixed lines as did sunlight, and in exactly the same places.

To observe the *spectra of the light of the fixed stars,* and at the same time to *determine the refrangibility of this light,* I prepared a short time ago a suitable apparatus specially adapted to this end, the telescope belonging to it having an objective of 4 inches' aperture. With it I have obtained many interesting results, although the experiments are still far from being completed. The flint-glass prism of this instrument has an angle of 37° 40′, and has the same diameter as the objective. The angle which the incident ray makes with the ray emerging from the prism is about 26°, so that, if the refrangibility of the light of one star were to differ only slightly from that of another, the difference could be very easily detected. In order to measure with certainty this difference, in case such is found, I added a second smaller telescope, which is fastened to the larger one, and cuts it at an angle of about 26°—*i.e.,* at the angle which the ray emerging from the prism makes with the incident ray. Two observers are needed : one observes the arrival of the star at the cross-hairs of the smaller telescope, which views the star directly, not through the prism ; the other observes, through the larger telescope, the arrival of a portion of the spectrum of the same star. This larger instrument has for this purpose a micrometer-screw, whose movable edge is so placed by the observer, by means of the screw, that at the instant when the star passes the cross-hair of the smaller telescope, viewed directly without the prism, one of the fixed lines of the spectrum coincides with the edge in the larger telescope. The micrometer is left unchanged, and the instrument is then pointed at another star, concerning which one wishes to know if its light has the same refrangibility. If, at the instant when this star passes the cross-hair of the smaller telescope, the same color of the spectrum or the same fixed line arrives at the edge of the micrometer in the larger instrument, the refrangibility of the two kinds of light are the same. Since two observers are necessary for these experiments, Herr Soldner had the kindness to make them with me. These experiments are, however, to be regarded as only just begun; and I must make some important modifications in the instrument in order to obtain still greater accuracy and to save time in the observations.

Up to the present we have found no *fixed star* whose light,

so far as its *refrangibility* is concerned, is sensibly different from that of the *planets*. When the fixed lines of the spectra are seen plainly, one can be certain with this instrument to 10 seconds; and when the fixed lines cannot be seen, one can still be certain for the orange light to $\frac{1}{2}$ minute. Since the total refraction through the prism is 26°, a difference amounting to $\frac{1}{9360}$ of the whole refraction could still be noticed with this instrument, a difference which even with the horizontal refraction in the atmosphere did not amount to $\frac{1}{4}$ second. Up to this time, as is well known, some astronomers have doubted whether the refraction tables for different stars should not be somewhat different; therefore this doubt seems to be removed by the experiment noted. The continuation of this investigation will lead us, I hope, to more complete knowledge.

In order to see the *fixed lines* of the different stars (with this large instrument) the air must be most favorable—a condition which happens rarely to a sufficient extent. The spectra of the light from *Mars* and *Venus* contain the same fixed lines as does sunlight, and in exactly the same places, at least so far as the lines D, E, *b*,* and F are concerned, whose relative positions can be exactly determined. In the spectrum of the light from *Sirius* I could not distinguish fixed lines in the orange and yellow; in the green, however, there is seen a very strong streak; and in the blue there are two other unusually strong streaks, which seem to be unlike any of the lines of planetary light. We have determined their positions with the micrometer. *Castor* gives a spectrum which is like that of Sirius; the streak in the green, in spite of the weak light, was intense enough for me to be able to measure it; and I found it in exactly the same place as it was with Sirius. I could also distinguish the streaks in the blue; but the light was too feeble to allow of measurement. In the spectrum of *Pollux* I recognized many fixed lines which resembled those of Venus; but all were weak. I saw the D line quite plainly, in exactly the same position as with planetary light. *Capella* gives a spectrum in which, at the places D and *b*, the same fixed lines are seen as in sunlight. The spectrum of *Betelgeux* [α Orionis] contains countless fixed lines which, with a good atmosphere, are sharply

* The line *b* lies in the green, between E and F; it consists of three strong lines, two of which are nearer than the third. See previous papers.

defined; and, although at first sight it seems to have no resemblance to the spectrum of Venus, yet similar lines are found in the spectrum of this fixed star in exactly the places where with sunlight D and *b* come. Some lines can be distinguished in the spectrum of *Procyon;* but they are seen with difficulty, and so indistinctly that their positions cannot be determined with certainty. I think I saw a line at the position D in the orange.

MUNICH, *July* 14, 1823.

JOSEPH FRAUNHOFER was born in Straubing, near Munich, March 6, 1787, and died in Munich, June 7, 1826. His father was a master glass-workman, and at the age of twelve he was apprenticed to a glass-polisher in Munich. In 1806 he became the optician in an optical-mechanical institute in Benediktbeuern. While there he invented new methods, machines, and measuring instruments to perfect the construction of achromatic lenses. In the course of this work he was led to the discovery of the lines in the solar spectrum which bear his name. With the aid of a prismatic-objective telescope he studied the spectra of the moon, the planets, and some of the stars. These results were presented to the Royal Academy of Sciences of Bavaria in 1814 and 1815, and were published in 1817, immediately after which he was elected a Member of the Academy. He returned to Munich to live in 1819, where he was engaged in the manufacture of optical instruments. In 1821 he published his experiments on diffraction through one or more openings, and on the spectra obtained by gratings. In 1823 this paper was continued by further experiments with glass gratings and by measurements of the wave-length of various solar lines. In 1825 appeared his memoir on halos, mock-suns, and allied phenomena. In the later years of his life he was appointed Royal Professor, and received numerous honors both from Bavaria and from foreign lands. He was buried in a plot of land given by the city of Munich for the purpose. His tomb bears the simple inscription, "*Approximavit sidera.*"

A Method of Examining Refractive and Dispersive Powers by Prismatic Reflection.

By WILLIAM HYDE WOLLASTON, M.D., F.R.S.

Read June 24, 1802. *Philosophical Transactions*, 1802.

[*A method is described which is based upon the principle of total reflection; and the results of experiments upon various solids and liquids are given. Then follows a brief discussion of the number of distinct colours in the solar spectrum. The author's ideas are based upon the following experiment.*]

If a beam of daylight be admitted into a dark room by a crevice $\frac{1}{20}$ of an inch broad, and received by the eye at a distance of 10 or 12 feet, through a prism of flint-glass, *free from veins,* held near the eye, the beam is seen to be separated into the four following colours only—red, yellowish green, blue, and violet, in the proportions represented in Fig. 3. [*Omitted.*]

The line A that bounds the red side of the spectrum is somewhat confused, which seems in part owing to want of power in the eye to converge red light. The line B, between red and green, in a certain position of the prism, is perfectly distinct; so also are D and E, the two limits of violet. But C, the limit of green and blue, is not so clearly marked as the rest, and there are also on each side of this limit other distinct dark lines—*f* and *g*—either of which, in an imperfect experiment, might be mistaken for the boundary of these colours.

The position of the prism in which the colours are most clearly divided is when the incident light makes about equal angles with two of its sides. I then found that the spaces AB, BC, CD, DE, occupied by them, were nearly as the numbers 16, 23, 36, 25.

Since the properties of these colours to each other have been supposed by Dr. Blair to vary according to the medium by

which they are produced, I have compared with this appearance the coloured images caused by prismatic vessels containing substances supposed by him to differ most in this respect, such as strong but colourless nitric acid, rectified oil of turpentine, very pale oil of sassafras, and Canada balsam, also nearly colourless. With each of these I have found the same arrangement of these four colours, and, in similar positions of the prisms, as nearly as I could judge, the same proportions of them.

But when the inclination of any prism is altered so as to increase the dispersion of the colours, the proportions of them to each other are then also changed, so that the spaces AC and CE, instead of being as before, 39 and 61, may be found altered as far as 42 and 58.*

By candlelight a different set of appearances may be distinguished. When a very narrow line of the blue light at the lower part of the flame is examined alone, in the same manner, through a prism, the spectrum, instead of appearing a series of lights of different hues contiguous, may be seen divided into five images at a distance from each other. The first is broad red, terminated by a bright line of yellow; the second and third are both green; the fourth and fifth are blue, the last of which appears to correspond with the division of blue and violet in the solar spectrum or the line D of Fig. 3.

When the object viewed is a blue line of electric light I have found the spectrum to be also separated into several images, but the phenomena are somewhat different from the preceding. It is, however, needless to describe minutely appearances which vary according to the brilliancy of the light, and which I cannot undertake to explain.

WOLLASTON was born in Norfolkshire, August 6, 1766, and died in London, December 22, 1828. He discovered the two elements palladium and rhodium, and invented a process for rendering platinum malleable. He is best known, perhaps, for his invention of the various instruments named after him—the reflecting goniometer, the cryophorus, a microscopic doublet.

* [*A note on the existence of the infra-red and the ultra-violet spectra.*]

BIBLIOGRAPHY

Spectroscopic Apparatus.

Michelson. Interference Methods.
Philosophical Magazine, (5), XXXI., 338–346, 1891; XXXIV., 280–299, 1892.
Astronomy and Astro-Physics, XI., 884–903, 1892.

Hale. Spectroheliograph.
Astronomy and Astro-Physics, XI., 407–417, 1892; XII., 241–257, 1893.

Pickering. Objective Prism.
Astronomy and Astro-Physics, XI., 199–203, 1892.

Hale and Wadsworth. Objective Prism.
Astrophysical Journal, IV., 54–79, 1896.

Wadsworth. Spectroscopes of Various Forms.
Astrophysical Journal, I., II., III., IV., 1895, 1896.

Solar Spectrum.

Kirchhoff und Bunsen. Poggendorff's *Annalen*, CX., CXI., CXIII., 1860, 1861.
Philosophical Magazine, (4), XX., 89, 1860.

Ångström. Recherches sur le Spectre Solaire, Upsala, 1868.

Rowland. *Astrophysical Journal*, I.–VI., 1895–1897.

Higgs. *Astrophysical Journal*, VII., 86, 1898.

Stellar Spectra.

See Scheiner, "Spectralanalyse der Gestirne," Braunschweig, 1883.
English Translation by Frost, Boston, 1894.

Spectra of the Elements.

See Scheiner, "Spectralanalyse."

See Kayser, "Spectralanalyse," Winkelmann's *Handbuch der Physik*, II.

See Watts, "Index of Spectra," Manchester.

Infra-Red Spectra.

H. Becquerel. *Annales Chimique et Physique*, (5), XXX., 1–144, 1883.

Langley. *Philosophical Magazine*, (5), XVII., XXI., XXII., XXVI., 1884–1888.
American Journal of Science, XXVII., XXXI., XXXII., XXXVI., 1884–1888.

PRISMATIC AND DIFFRACTION SPECTRA

Abney. *Philosophical Transactions*, p. 457, 1886.
Lewis. *Astrophysical Journal*, II., 1–25, 106–109, 1895.
Rubens and Nichols. Wiedemann's *Annalen*, LX., (3), 418–462, 1897.
Physical Review, IV., 314–323, 1897.

Ultra-Violet Spectra.

Schumann. *Anzeige der Wiener Akademie*, XI., 1895.
Astrophysical Journal, III., 220, 387, 1896.

Standard Wave-Lengths.

Rowland. *Philosophical Magazine*, (5), XXXVI., 49–75, 1893.
Astronomy and Astro-Physics, XII., 321–346, 1893.
Michelson. *Mémoires du Bureau International des Poids et Mesures*, XI., 1894.
Bell. *Philosophical Magazine*, (5), XXV., 245, 350, 1888.

Origin of Fraunhofer Lines.

Kirchhoff. *Abhandlungen der Berliner Akademie*, 1861.
Poggendorff's *Annalen*, CIX., 275, 1860.
Philosophical Magazine, (4), XX., 1860.
Rowland. Johns Hopkins University *Circulars*, 10, 1891.
Janssen. *Comptes Rendus*, 56, 1860; 60, 1864; 63, 1866; 78, 1874; 101, 1885; 102, 1886; 106, 1888; 107, 1888; 108, 1889.

Diffraction Gratings and their Adjustment.

Rowland. Gratings in Theory and Practice.
Astronomy and Astro-Physics, XII., 129–149, 1893.
Philosophical Magazine, (5), XXXV., 397, 1893.
Concave Gratings for Optical Purposes.
American Journal of Science, (3), XXVI., 1883.
Philosophical Magazine, (5), XVI., 197, 1883.
Rayleigh. "Wave-Theory of Light." *Encyclopedia Britannica.*
Manufacture and Theory of Diffraction Gratings.
Philosophical Magazine, (4), XLVII., 1874.
Kayser. "Spectralanalyse." Winkelmann, *Handbuch der Physik*, Band II., Abth. I., 403–413, 1894.
Wadsworth. The Modern Spectroscope.
Astrophysical Journal, I., II., III., 1895, 1896.
Ames. Concave Grating in Theory and Practice.
Philosophical Magazine, (5), XXVII., 1889.
Astronomy and Astro-Physics, XI., 28–42, 1892.

INDEX

INDEX

THE END

G

[illegible]